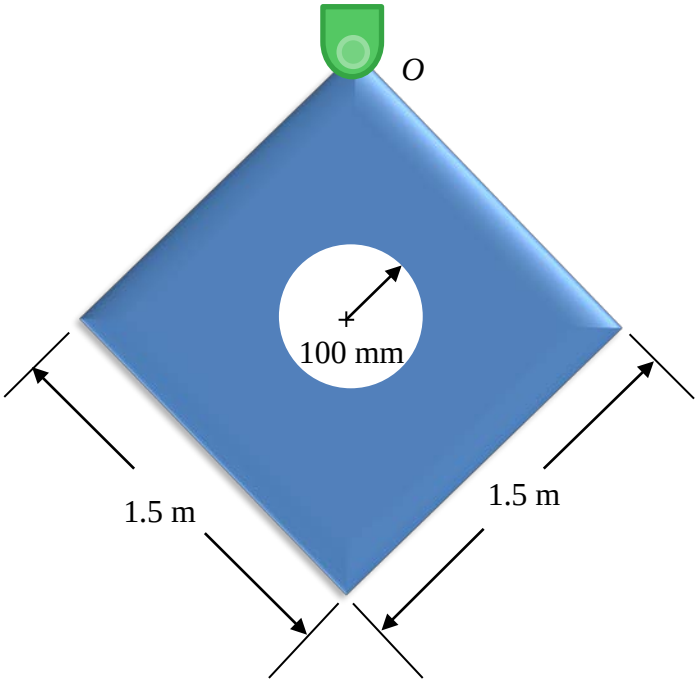
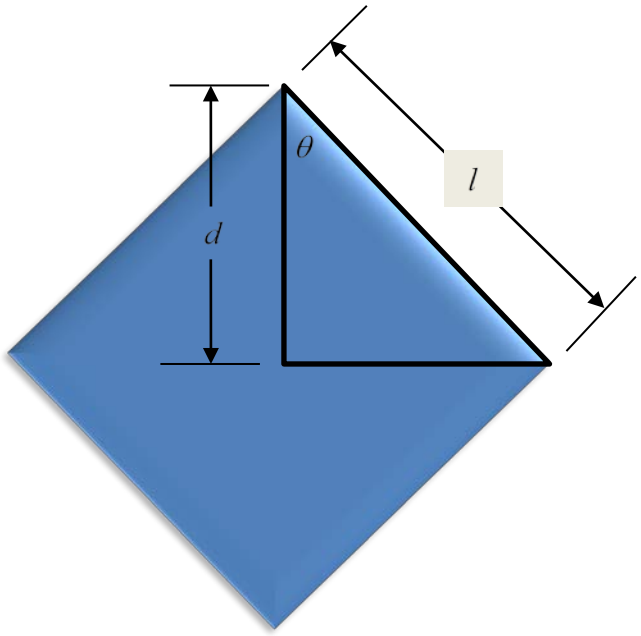
4-5. A thin plate with a hole at its center is hanging from the pivot point *O* by one of its corners. It has a thickness of $t \sim N(40,3^2)$ mm and the density of the material is $\rho = 30$ kg/m^3. Find the moment of inertia about the axis that passes through *O*, perpendicular to the page.



Solution:



$$d = l \sin\theta = 1.5 \sin 45°$$

$$I_O = \Sigma I_G + md^2$$

$$I_G = \frac{1}{12}(\rho l^2)(l^2 + l^2)t - \frac{1}{2}(\rho\pi r^2)(r^2)t = \left(\frac{1}{6}\rho l^4 - \frac{1}{2}\rho\pi r^4\right)t$$

$$m = (\rho l^2 - \rho\pi r^2)t$$

$$I_O = I_G + md^2 = \left(\frac{1}{6}\rho l^4 - \frac{1}{2}\rho\pi r^4\right)t + (\rho l^2 - \rho\pi r^2)td^2$$

$$= \left[\left(\frac{1}{6}\rho l^4 - \frac{1}{2}\rho\pi r^4\right) + (\rho l^2 - \rho\pi r^2)d^2\right]t$$

thus,

$$\mu_{I_O} = \left[\left(\frac{1}{6}\rho l^4 - \frac{1}{2}\rho\pi r^4\right) + \left(\rho l^2 - \rho\pi r^2\right)d^2\right]\mu_t = 4.01 \text{ kg} \cdot \text{ m}^2$$

$$\sigma_{I_O} = \sqrt{\left[\left(\frac{1}{6}\rho l^4 - \frac{1}{2}\rho\pi r^4\right) + \left(\rho l^2 - \rho\pi r^2\right)d^2\right]^2 \sigma_t^2} = 0.30 \text{ kg} \cdot \text{ m}^2$$

Therefore, $I_O \sim N(4.01, 0.30^2)$ kg · m^2 **Ans.**