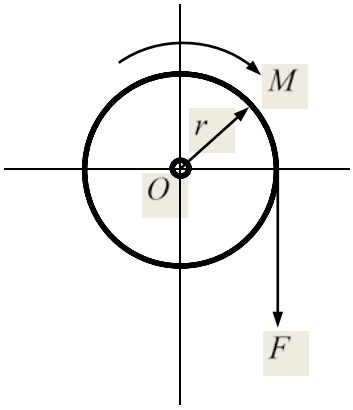
4-8. The 20 kg disk is subjected to a normally distributed couple moment $M \sim N(10, 1^2)\ \text{N}\cdot\text{m}$. A force $F \sim N(5, 0.5^2)\ \text{N}$ is applied to a cord wrapped around its periphery. The radius of the disk is $r = 0.5\ \text{m}$. Determine the angular velocity of the disk 2 seconds after starting from rest. Assume F and M are independent.



$$I_O = \frac{1}{2}mr^2$$

$$I_O w_1 + \sum \int_{t_1}^{t_2} M_O dt = I_O w_2$$

$$0 + Mt + Frt = \frac{1}{2}mr^2 w_2$$

$$w_2 = \frac{2(Mt + Frt)}{mr^2}$$

$$\mu_{w_2} = \frac{2(\mu_M t + \mu_F rt)}{mr^2} = \frac{2(10(2) + 5(0.5)(2))}{20(0.5)^2} = 10\ \text{rad/s}$$

$$\sigma_{w_2} = \sqrt{\left(\frac{2\sigma_M t}{mr^2}\right)^2 + \left(\frac{2\sigma_F rt}{mr^2}\right)^2} = \sqrt{\left(\frac{2(1)(2)}{20(0.5)^2}\right)^2 + \left(\frac{2(0.5)(0.5)(2)}{20(0.5)^2}\right)^2} = 0.82\ \text{rad/s}$$

Therefore, $w_2 \sim N(10, 0.82^2)\ \text{rad/s}$. **Ans.**