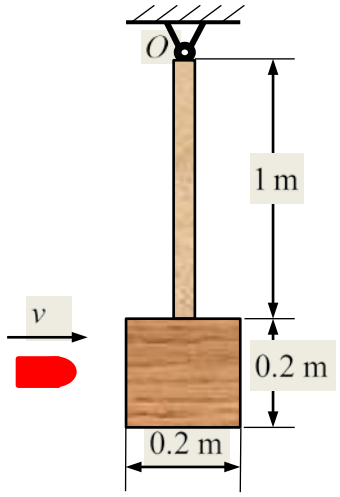
4-9. A pendulum is initially at rest. It consists of a slender rod and a wooden block. A bullet with a normally distributed velocity of $v \sim N(1000, 100^2)$ m/s hits the block and embeds itself into the center of block. If the masses of the rod, block and bullet are $m_1 = 0.2$ kg, $m_2 = 1$ kg and $m_3 = 5$ g, respectively, determine the probability that the angular velocity is smaller than 4 rad/s just after the impact.



$$(I_O)_2 = \frac{1}{12}m_1(1^2) + m_1\left(\frac{1}{2}\right)^2 + \frac{1}{12}m_2(0.2^2 + 0.2^2) + m_2\left(1+\frac{0.2}{2}\right)^2 + m_3\left(1+\frac{0.2}{2}\right)^2$$

$$= \frac{1}{3}m_1 + 1.22m_2 + 1.21m_3 = \frac{1}{3}(0.2) + 1.22(1) + 1.21(0.005)$$

$$= 1.29 \text{ kg}\cdot\text{m}^2$$

$$(H_O)_1 = (H_O)_2$$

$$m_3 v l = (I_O)_2 w_2$$

$$w_2 = \frac{1.1 m_3 v}{1.29}$$

$$\mu_{w_2} = \frac{1.1 m_3 \mu_v}{1.29} = \frac{1.1(0.005)(1000)}{1.29} = 4.26 \text{ rad/s}$$

$$\sigma_{w_2} = \frac{1.1 m_3 \sigma_v}{1.29} = \frac{1.1(0.005)(100)}{1.29} = 0.43 \text{ rad/s}$$

$$\Pr\{w_2 < 4\} = \Phi\left(\frac{4-\mu_{w_2}}{\sigma_{w_2}}\right) = \Phi\left(\frac{4-4.26}{0.43}\right) = 0.27$$