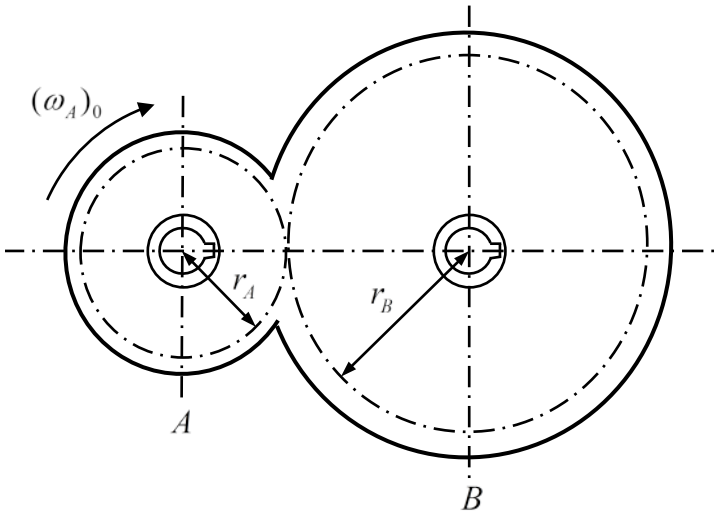
3-9. Gear A has a normally distributed initial angular velocity $(\omega_A)_0 \sim N(5, 0.5^2)$ rad/s, then it accelerates with the angular acceleration $\alpha = (3t^2)$ rad/s^2, where t is in seconds. If $r_A = 0.2$ m and $r_B = 0.6$ m, determine the probability that the angular velocity of gear B is smaller than 4.5 rad/s when $t = 2$ s.



$$\omega_A = (\omega_A)_0 + \int_0^t \alpha dt = (\omega_A)_0 + \int_0^t 3t^2 dt = (\omega_A)_0 + t^3$$

$$\omega_A r_A = \omega_B r_B$$

$$\omega_B = \frac{\omega_A r_A}{r_B} = \frac{r_A t^3}{r_B} + \frac{r_A(\omega_A)_0}{r_B}$$

$$\mu_{\omega_B} = \frac{r_A t^3}{r_B} + \frac{r_A \mu_{(\omega_A)_0}}{r_B} = \frac{0.2(2^3)}{0.6} + \frac{0.2(5)}{0.6} = 4.33 \text{ rad/s}$$

$$\sigma_{\omega_B} = \frac{r_A \sigma_{(\omega_A)_0}}{r_B} = \frac{0.2(0.5)}{0.6} = 0.17 \text{ rad/s}$$

$$\Pr(\omega_B < 4.5) = \Phi\left(\frac{4.5 - \mu_{\omega_B}}{\sigma_{\omega_B}}\right) = \Phi\left(\frac{4.5 - 4.33}{0.17}\right) = 0.84$$