Deal with Uncertainty in Dynamics

Missouri S&T

Founded 1870 | Rolla, Missouri

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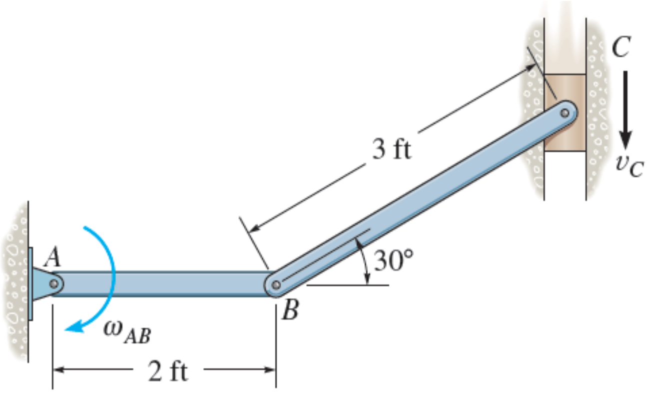
Outline

- Uncertainty in dynamics
- Why consider uncertainty
- Basics of uncertainty
- Uncertainty analysis in dynamics
- Examples
- Applications

Uncertainty in Dynamics

- Example: given $\omega_{AB} = \text{const} = 2 \text{ rad/s}$, find v_c and a_c
- We found $v_c = 4 \text{ ft/s}$ and $a_c = 13.86 \text{ ft/s}^2$
- Everything is modeled perfectly.
- In reality, ω_{AB}, l_{AB}, and l_{BC} are all random.
- So are the solutions.
  - v_c will fluctuate around 4 ft/s.



Where Does Uncertainty Come From?

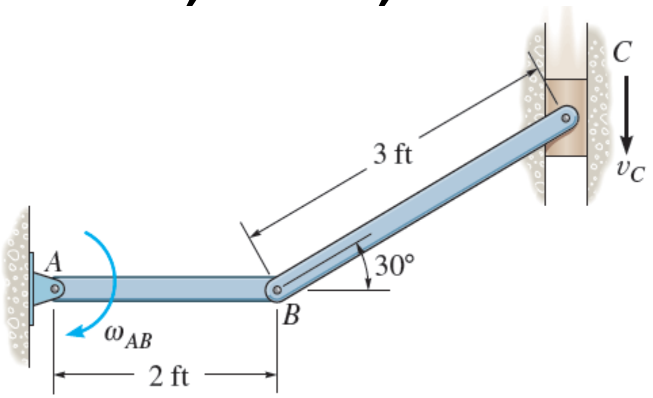
- Manufacturing impression
  - Dimensions of a mechanism
  - Material properties
- Environment
  - Loading
  - Temperature
  - Different users

Why Consider Uncertainty?

- We know the true solution.
- We know the effect of uncertainty.
- We can make more reliable decisions.
  - If we know uncertainty in the traffic to the airport, we can better plan our trip and have lower chance of missing flights.

How Do We Model Uncertainty?

- Measure $X = \omega_{AB}$ ten times, we get
- $X =$ (2.17, 1.82, 2.0, 1.89, 2.06, 1.88, 2.10, 2.15) rad/s



- How do we use the samples?
- Average $\mu = \frac{1}{10}(2.17+1.82+\cdots+2.10+2.15)=\frac{1}{10}\sum X_i = 2.04$ rad/s (use Excel)

How Do We Measure the Dispersion?

- $X = (2.17, 1.82, 2.0, 1.89, 2.06, \; 1.88, 2.10, 2.15)$
- We could use $X_i - \mu$ and $\frac{1}{N}\sum(X_i - \mu)$, $N = 10$
- But $\frac{1}{N}\sum(X_i - \mu)$=0.
- To avoid 0, we use $\frac{1}{N}\sum(X_i - \mu)^2$; to have the same unit as

  $\bar{X}$, we use $\sigma = \sqrt{\frac{1}{N}\sum(X_i - \mu)^2}$
- We actually use

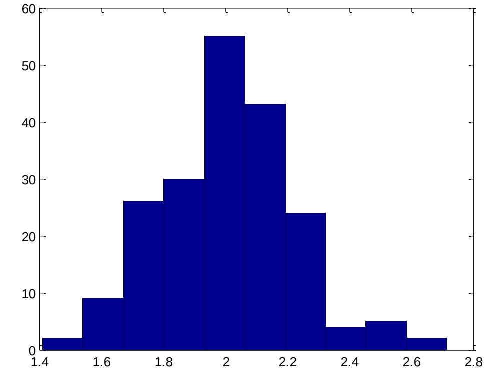
  Standard deviation: $\sigma = \sqrt{\frac{1}{N-1}\sum(X_i - \mu)^2}$.
- Now we have σ =0.16 rad/s for $\mu = 2.04$ rad/s. (Use Excel)

More about Standard Deviation (std)

- It indicates how data spread around the mean.
- It is always non-negative.
- High std means
  - High dispersion
  - High uncertainty
  - High risk

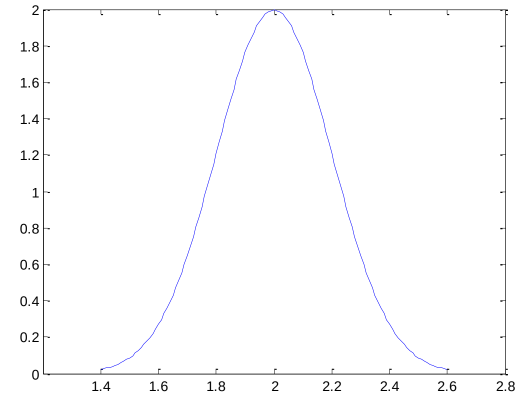
Probability Distribution

- With more samples, we can draw a histogram.



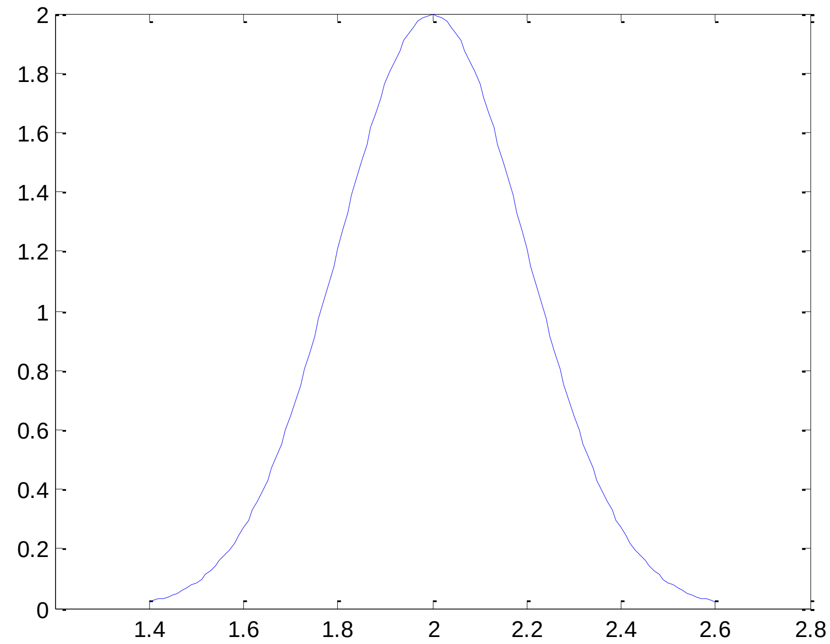
- If y-axis is frequency and the number of samples is infinity, we get a probability density function (PDF) $f(x)$.
- The probability of $a \leq X \leq b$.

$$\Pr\{a \leq X \leq b\} = \int_a^b f(x)dx$$



Normal Distribution

- $X \sim N(\mu, \sigma^2)$
- $F(x) = \Pr\{X < x\}$ is called cumulative distribution function (CDF)
- $\Pr\{a < X < b\} = F(b) - F(a)$
- $\Pr\{X > x\} = 1 - \Pr\{X < x\} = 1 - F(x)$



How to Calculate $F(x)$?

- Use Excel
  - NORMDIST(x,mean,std,cumulative)
  - cumulative=true

Example 1

- Average speeds (mph) of 20 drivers from S&T to STL airport were recoded.
- The average speed is normally distributed.
  - Mean =? Std = ?
  - What is the probability that a person gets to the airport in 2 hrs if d = 120 mile?

1	62.2
2	58.2
3	70.9
4	64.9
5	68.3
6	67.9
7	67.1
8	63.0
9	63.2
10	65.5
11	62.3
12	62.8
13	72.9
14	67.2
15	62.1
16	66.8
17	62.5
18	61.6
19	66.0
20	70.8

Solution: Use Excel

- X = average speed
- $X \sim N(\mu, \sigma^2)$
  - $\mu = 65.3$ mph by AVERAGE
  - σ= 3.72 mph by STDEV
- For $t \leq 2$ hr, $X \geq \frac{120}{2} = 60$ mph
- $\Pr\{X \geq 60\} = 1 - \Pr\{X < 60\}$

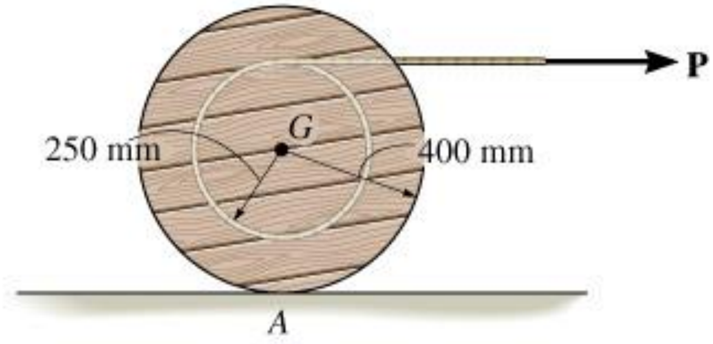
$= 1 -$ NORMDIST(60,65.3,3.72,true)=0.92

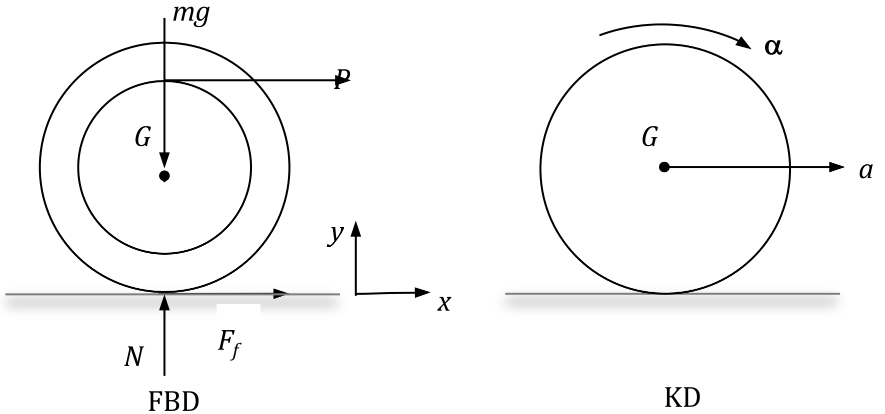
More Than One Random Variables

- $X_i \sim N(\mu_i, \sigma_i^2)$
- $Y = c_0 + c_1 X_1 + c_2 X_2 + \cdots + c_n X_n$
- $c_i \ (i = 1,2,\cdots,n)$ are constants.
- Then $Y \sim N(\mu_Y, \sigma_Y^2)$
- $\mu_Y = c_0 + c_1\mu_1 + c_2\mu_2 + \cdots + c_n\mu_n$
- $\sigma_Y = \sqrt{c_1^2\sigma_1^2 + c_2^2\sigma_2^2 + \cdots + c_n^2\sigma_n^2}$

Example 2

- The spool has a mass of m = 100 kg and a radius of gyration $k_g = 0.3\ m$. It is subjected to force $P = 1000\ \mathrm{N}$. If the coefficient of static friction at A is $\mu_s = 0.15$, determine if the spool slips at point A.





Assume no slipping: $a = r\alpha$ (1)

$$\Sigma F_x = ma_{Gx} : P + F_f = ma \quad (2)$$

$$\Sigma F_y = ma_{Gy} : N - mg = 0 \quad (3)$$

$$\Sigma M_G = I_G\alpha = Pr_p - F_f r = mR_g^2\alpha \quad (4)$$

where r_p=0.25 m, r=0.4 m

Solving (1)—(4):

$$F_f = P\left[\frac{r(r+r_p)}{R_g^2+r^2} - 1\right] = 1000\left[\frac{0.4(0.4+0.25)}{0.3^2+0.4^2} - 1\right] = 40.0N$$

($F_f = c_2 P$ where c_2=0.04; we'll use it later)

N=981.0 N

$F_{max} = \mu_s N$=0.15(981.0)=147.15 N

($F_{max} = c_1 \mu_s$ where c_1=981; we'll use it later)

F_f=40.0 $N <$ $maxF$ =147.15N

The assumption is OK, and the spool will not slip.

When Consider Uncertainty

- If $\mu_s \sim \mathrm{N}(\mu_1, \sigma_1^2) = \mathrm{N}(0.15, 0.03^2)$

  $P \sim \mathrm{N}(\mu_2, \sigma_2^2) = \mathrm{N}(1000, 140^2)\ N$
- What is the probability that the spool will slip ?

Let $Y = F_{max} - F_f = c_1 \mu_s - c_2 P$

If $Y < 0$, the spool will slip.

$$\mu_Y = \mu_{F_{max}} - \mu_{F_f} = 147.15 - 40 = 107.15\text{N}$$

$$\sigma_Y = \sqrt{c_1^2 \sigma_{F_{max}}^2 + c_2^2 \sigma_{F_\mu}^2} = \sqrt{981^2(0.03^2) + (-0.04)^2(140^2)}$$

$$= 29.9581 \text{ N}$$

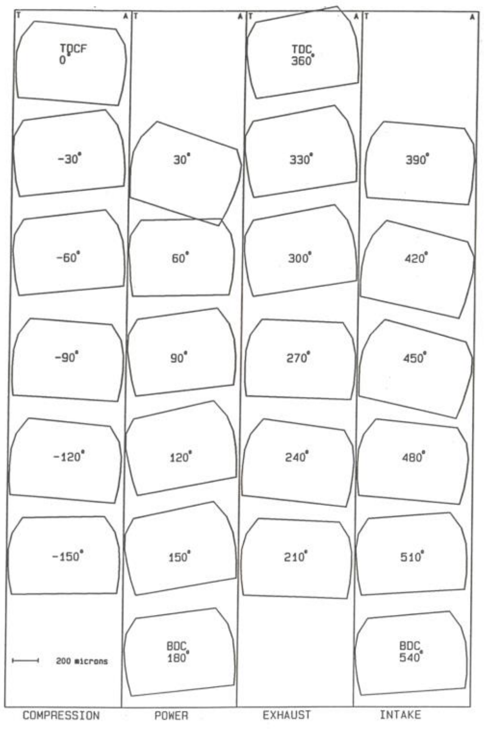
Pr{Slipping} = Pr{Y<0}=F(0, 107.15, 29.9581, True)

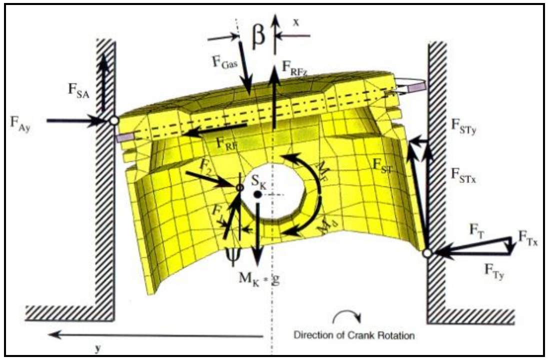
$=1.74\times10^{-4}$ (Use Excel)

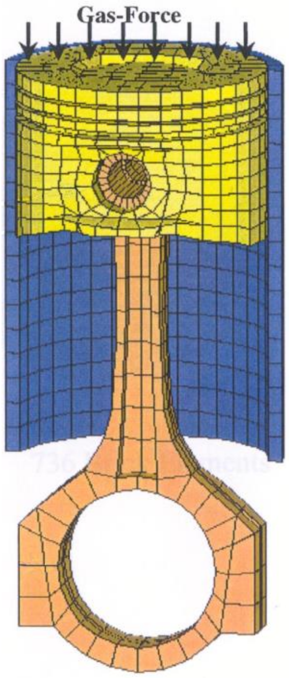
Engine Robust Design

$$\text{minimize} \quad f(\mu_{noise}, \ \sigma_{noise})$$

Piston (Skirt) Motion Summary

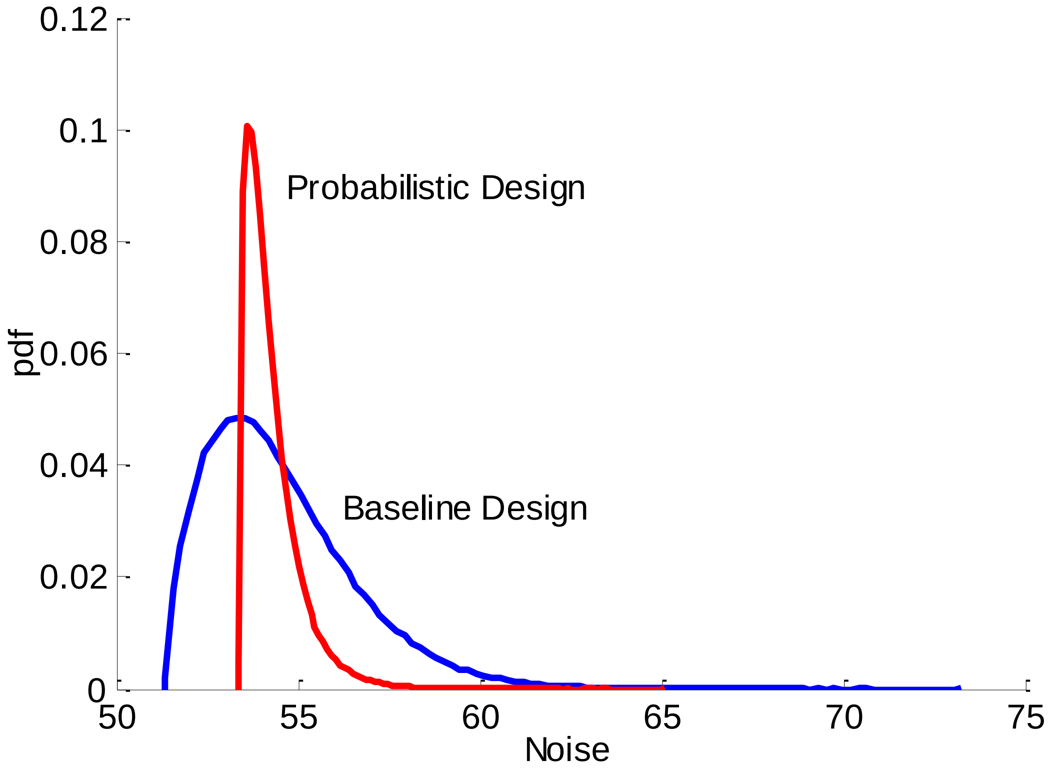






Result Analysis

- The mean noise reduced by 0.5%.
- The standard deviation reduced by 62.5%.



Assignment

- On Blackboard.