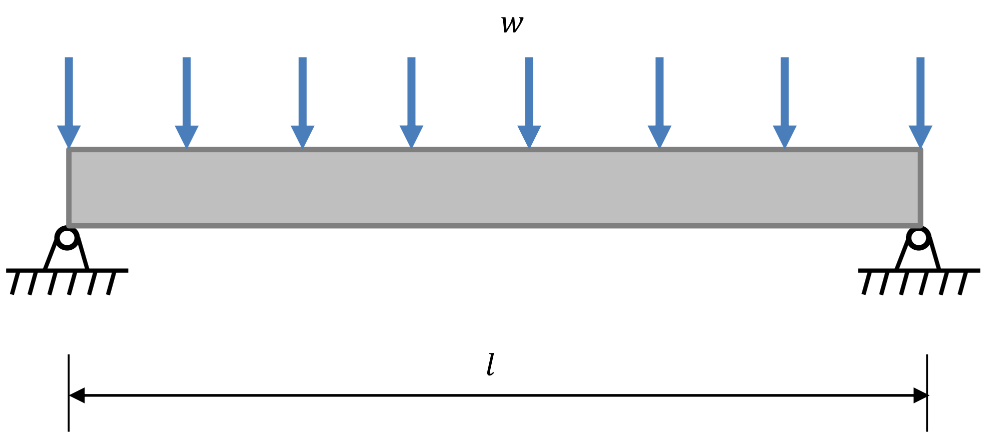
64. A square-cross-section beam is simply-supported and is subjected to an uniform load of $w \sim N(8000, 800^2)$ N/m as depicted in the figure. The length of the beam is $l = 3.5$ m and the side length of cross section is $b = 50$ mm. If the allowable bending stress is $S_a \sim N(50, 5^2)$ MPa, estimate the probability of failure using the First Order Second Moment Method. Assume that w and S_a are independent.



Solution

The reaction force is

$$R_1 = \frac{wl}{2}$$

The maximum bending moment occurs at $x = \frac{l}{2}$ and is given by

$$M = R_1 \frac{l}{2} - w \frac{l}{4} = \frac{wl^2}{8}$$

Thus the bending stress is

$$S = \frac{Mc}{I} = \frac{\frac{wl^2}{8}\frac{b}{2}}{\frac{bb^3}{12}} = \frac{3wl^2}{4b^3}$$

The limit-state function is the actual bending stress subtracted from the allowable one. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = S_a - S = S_a - \frac{3wl^2}{4b^3}$$

where $\mathbf{X}$=(S_a, w).

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_{\mathbf{X}}) = \mu_{S_a} - \frac{3\mu_w l^2}{4b^3} = 50(10^6) - \frac{3(8000)3.5^2}{4\left(50(10^{-3})\right)^3} = 2.06(10^7) \quad \text{Pa}$$

$$\sigma_Y = \sqrt{\left(\left.\frac{\partial g}{\partial S_a}\right|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_{S_a}\right)^2 + \left(\left.\frac{\partial g}{\partial w}\right|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_w\right)^2}$$

$$= \sqrt{\left(\sigma_{S_a}\right)^2 + \left(-\frac{3l^2}{4b^3}\sigma_w\right)^2}$$

$$= \sqrt{\left(5(10^6)\right)^2 + \left(-\frac{3(3.5^2)}{4\left(50(10^{-3})\right)^3}(800)\right)^2}$$

$$= 5.8003(10^6) \qquad \text{Pa}$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(-\frac{2.06(10^7)}{5.8003(10^6)}\right) = 1.92(10^{-4})$$

Ans.