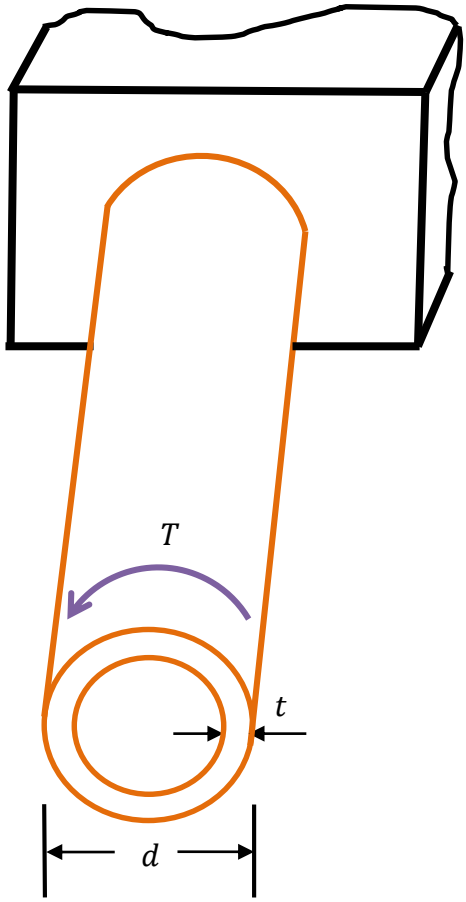
68. A torque $T \sim N(1000, 100^2)$ N·m is applied to a welded thin-wall steel tube as shown in the figure. The tube is of round section with a diameter of $d = 40$ mm. It has a a thickness of $t = 5$ mm and a length of $l \sim N(500, 0.1^2)$ mm. The shear modulus is $G = 80$ GPa. If the allowable anlge of twist is $\theta_a = 0.05$, estimate probability of failure using the First Order Second Moment Method. Note that T and θ_a are independent.



Solution

Based on the theory of closed thin-walled tube, the angular twist per unit of length is

$$\theta_1 = \frac{TL_m}{4GA_m^2 t}$$

where $L_m = \pi(d - t)$ is the section median line and A_m is the area enclosed by section median line.

$$A_m = \frac{\pi}{4}(d - t)^2$$

Thus the actual angle of twist is

$$\theta = \theta_1 l = \frac{L_m}{4GA_m^2 t} Tl = \frac{\pi(d-t)}{4G\left(\frac{\pi}{4}(d-t)^2\right)^2 t} Tl = \frac{4}{G\pi(d-t)^3 t} Tl$$

The limit-state function is the actual angle of twist subtracted from the allowable one. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = \theta_a - \theta = \theta_a - \frac{4}{G\pi(d-t)^3 t} Tl$$

where $\mathbf{X}=(T, l)$.

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_\mathbf{X}) = \theta_a - \frac{4}{G\pi(d-t)^3 t}\mu_T \mu_l$$

$$= 0.05 - \frac{4}{(80)(10^9)\pi\left(40(10^{-3}) - 5(10^{-3})\right)^3 8(10^{-3})} 1000(500)(10^{-3})$$

$$= 1.2879(10^{-2})$$

$$\sigma_Y = \sqrt{\left(\left.\frac{\partial g}{\partial T}\right|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_T\right)^2 + \left(\left.\frac{\partial g}{\partial l}\right|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_l\right)^2}$$

$$= \sqrt{\left(-\frac{4\mu_l}{G\pi(d-t)^3 t}\sigma_T\right)^2 + \left(-\frac{4\mu_T}{G\pi(d-t)^3 t}\sigma_l\right)^2}$$

$$= \sqrt{\begin{array}{l}\left(\dfrac{4(500)(10^{-3})}{(80)(10^9)\pi\left(40(10^{-3}) - 5(10^{-3})\right)^3 8(10^{-3})} 100\right)^2 \\ +\left(\dfrac{4(1000)}{(80)(10^9)\pi\left(40(10^{-3}) - 5(10^{-3})\right)^3 8(10^{-3})} (0.1)(10^{-3})\right)^2\end{array}}$$

$$= 3.7121(10^{-3})$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-1.2879(10^{-2})}{3.7121(10^{-3})}\right) = 2.61(10^{-4})$$

Ans.