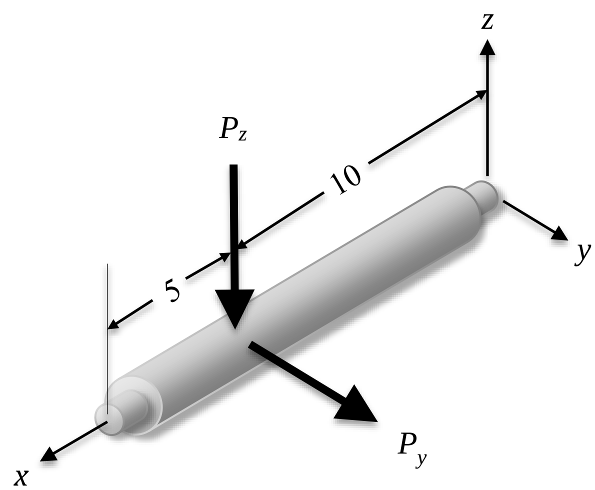
1. A simply-supported steel beam has a diameter of 1.25 in, and design requirements demand that the deflection in any direction at $x = 10$ in should be less than $\delta = 0.00375$ in. Find the probability of failure given that $P_z \sim N(100,8^2)$ lb and $P_y \sim N(200,10^2)$ lb using Monte Carlo Simulation. Note that P_z and P_y are independent.



Solution

Step 1: Define the limit-state function.

$$\delta_y = \frac{-P_y ab}{6EIL}(a^2 + b^2 - L^2)$$

$$\delta_z = \frac{P_z ab}{6EIL}(a^2 + b^2 - L^2)$$

$$\delta = \sqrt{{\delta_y}^2 + {\delta_z}^2} = \sqrt{\left[\frac{-P_y ab}{6EIL}(a^2 + b^2 - L^2)\right]^2 + \left[\frac{P_z ab}{6EIL}(a^2 + b^2 - L^2)\right]^2}$$

Limit-state function is then given by

$$g(\mathbf{X}) = \delta_{max} - \delta \text{ , where } \mathbf{X} = (P_y, P_z).$$

Therefore,

$$g(\mathbf{X}) = \delta_{max} - \sqrt{\left[\frac{-P_y ab}{6EIL}(a^2 + b^2 - L^2)\right]^2 + \left[\frac{P_z ab}{6EIL}(a^2 + b^2 - L^2)\right]^2}$$

where a is the distance to forces from origin, L is the length of the beam, $b = L - a$, E is the Young's modulus of steel, and I is the moment of inertia about y or z axis.

Step 2: Sample the random variables.

Step 3: Evaluate the limit-state function.

Step 4: Analyze statistically.

Failure occurs when $g(\mathbf{X}) < 0$.

Define an indicator function

$$\mathrm{I}(\mathbf{X}) = \begin{cases} 1 & \text{if } g(\mathbf{X}) < 0 \\ 0 & \text{otherwise} \end{cases}$$

Sum the number of failures

$$N_f = \mathrm{sum}(\mathrm{I}(\mathbf{X}))$$

Probability of failure

$$p_f = \frac{N_f}{N}$$

Number of Failures: 248092

Number of simulations: 1.000e+07

Probability of failure: 0.0248092 **Ans.**