10. A grinding wheel has a diameter of $d_o = 250$ mm, a thickness of $t = 5$ mm, and a bore with a diameter of $d_i = 20$ mm. The speed of the wheel is $n = 2000$ rev/min. The material is isotropic, and the Poisson's ration is $\nu = 0.20$. If the weight of the wheel is $m \sim N(0.8,\ 0.1^2)$ kg, the allowable tangential stress is $S_a \sim N(3,\ 0.2^2)$ MPa, and m and S_a are independent, determine the probability of failure using the First Order Second Moment Method.

Solution

The angular velocity of the wheel is

$$\omega = \frac{2\pi n}{60} = \frac{2\pi(2000)}{60} = 209.44 \text{ rad/s}$$

The outside radius and inside radius of the wheel are

$$r_o = \frac{d_o}{2} = \frac{250}{2} = 125 \text{ mm}$$

$$r_i = \frac{d_i}{2} = \frac{20}{2} = 10 \text{ mm}$$

The mass density of the wheel is

$$\rho = \frac{m}{V} = \frac{m}{\frac{\pi}{4}(d_o^2 - d_i^2)t} = 4100.61m$$

Based on the theory of rotating rings, the maximum tangential stress is

$$S_{max} = \rho\omega^2\left(\frac{3+\nu}{8}\right)\left(r_i^2 + r_o^2 + \frac{r_i^2 r_o^2}{r_i^2} - \frac{1+3\nu}{3+\nu}r_i^2\right)$$

$$= 4100.61m(209.44^2)\left(\frac{3+0.20}{8}\right)\left(0.01^2 + 0.125^2 + 0.125^2 - \frac{1+3(0.20)}{3+0.20}(0.01^2)\right)$$

$$= 2252019m$$

The limit-state function is the maximum tangential stress of the wheel subtracted from the allowable tangential stress. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = S_a - S_{max} = S_a - 2252019m$$

where $\mathbf{X}$=(S_a, m).

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_\mathbf{X}) = \mu_{S_a} - 2252019\mu_m = 3(10^6) - 2252019(0.8) = 1.20(10^6) \text{ psi}$$

$$\sigma_Y = \sqrt{\left(\left.\frac{\partial g}{\partial S_a}\right|_{\mu_X} \sigma_{S_a}\right)^2 + \left(\left.\frac{\partial g}{\partial m}\right|_{\mu_X} \sigma_m\right)^2} = \sqrt{\left(\sigma_{S_a}\right)^2 + (-2252019\sigma_m)^2}$$

$$= \sqrt{(0.2(10^6))^2 + (-2252019(0.1))^2} = 3.01(10^5)\ \text{psi}$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_g}{\sigma_g}\right) = \Phi\left(\frac{-1.20(10^6)}{3.01(10^5)}\right) = 3.4628(10^{-5})$$ **Ans.**