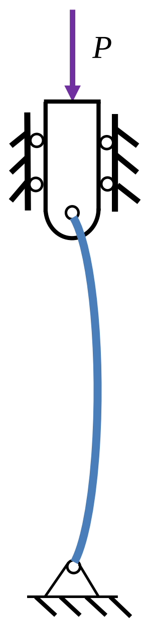
11. A round column is 1.5 m long and is subjected to an axial force $P \sim N(80,\ 8^2)$ kN. The ends are pinned as shown in the figure, and the modulus of elasticity is $E = 200$ GPa. If the diameter of the column is $d \sim N(40,\ 0.1^2)$ mm, and P and d are independent, determine the probability of bending failure using the First Order Second Moment Method.



Solution

Based on the theory of Euler column formula, the critical load for unstable bending is

$$P_{cr} = \frac{C\pi^2 EI}{l^2} = \frac{C\pi^2 E}{l^2}\frac{\pi d^4}{64} = \frac{CE\pi^3 d^4}{64 l^2}$$

where $C = 1$ is a constant, depending on the end conditions shown in the figure, I is the moment of inertia, and l is the length of the column.

The limit-state function is the actual load of the column subtracted from the critical load. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = P_{cr} - P = \frac{CE\pi^3 d^4}{64 l^2} - P = 4.31(10^{10})d^4 - P$$

where $\mathbf{X}$=(d, P).

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_{\mathbf{X}}) = 4.31(10^{10})\mu_d^4 - \mu_P = 4.31(10^{10})\left(40(10^{-3})\right)^4 - 80(10^3) = 3.02(10^4) \text{ N}$$

$$\sigma_Y = \sqrt{\left(\left.\frac{\partial g}{\partial d}\right|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_d\right)^2 + \left(\left.\frac{\partial g}{\partial P}\right|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_P\right)^2} = \sqrt{(4.31(10^{10})(4)\mu_d^3\sigma_d)^2 + (-\sigma_P)^2}$$

$$= \sqrt{\left(4.31(10^{10})(4)\left(40(10^{-3})\right)^3(0.1)(10^{-3})\right)^2 + (-8(10^3))^2} = 8.08(10^3)\ \text{N}$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_g}{\sigma_g}\right) = \Phi\left(\frac{-3.02(10^4)}{8.08(10^3)}\right) = 9.0135(10^{-5})$$ **Ans.**