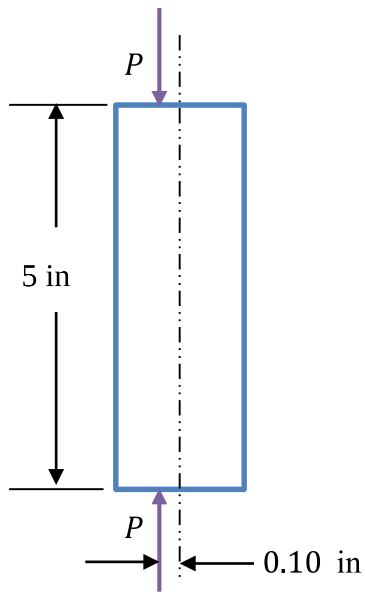
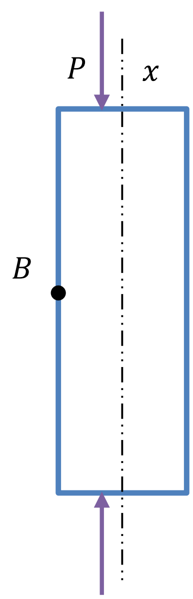
12. The size of a strut is $1 \times 1 \times 5$ in. The strut is subjected to an eccentrial force $P \sim N(800,\ 8^2)$ lbf with an offset of $e = 0.10$ in as shown in the figure. If the yield strength of the strut is $S_y \sim N(2200,\ 220^2)$ psi and P and S_y are independent, determine the probability of failure using the First Order Second Moment Method.



Solution



According to the theory of short compression member, the maximum compressive stress occurs at point B, and its magnitude is

$$S_{max} = \frac{P}{A}\left(1 + \frac{ecA}{I}\right)$$

where A is the cross-section area of the strut, c is the distance between point B and axis x, and I is the moment of inertia.

The limit-state function is the maximum compression stress of the strut subtracted from the yield strength. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = S_y - S_{max} = S_y - \frac{P}{A}\left(1 + \frac{ecA}{I}\right) = S_y - \frac{P}{1(1)}\left(1 + \frac{0.1(0.5)1(1)}{\frac{1(1)^3}{12}}\right) = S_y - 1.6P$$

where $\mathbf{X}=(S_y, P)$.

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_{\mathbf{X}}) = \mu_{S_y} - 1.6\mu_P = 2200 - 1.6(800) = 920 \text{ psi}$$

$$\sigma_Y = \sqrt{\left(\left.\frac{\partial g}{\partial S_y}\right|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_{S_y}\right)^2 + \left(\left.\frac{\partial g}{\partial P}\right|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_P\right)^2} = \sqrt{\left(\sigma_{S_y}\right)^2 + (-1.6\sigma_P)^2}$$

$$= \sqrt{(220)^2 + (-1.6(8))^2} = 220.37 \text{ psi}$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_g}{\sigma_g}\right) = \Phi\left(\frac{-920}{220.37}\right) = 1.4915(10^{-5})$$ **Ans.**