15. A reversing nominal stress $S \sim N(100,\ 10^2)$ kpsi with 500 cylces is applied to a rotating-beam. The fatigue strength fraction is $f = 0.8$. If the ultimate strength of the beam is $S_{ut} \sim N(220,\ 20^2)$ kpsi, and S and S_{ut} are independent, determine the probability of failure using the First Order Second Moment Method.

Solution

The failure strength is given by

$$S_f = S_{ut} N^{(\log f)/3}$$

So the limit-state function is the reversing nominal stress subtracted from the failure strength. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = S_f - S = S_{ut} N^{\frac{\log f}{3}} - S = S_{ut} 500^{\frac{\log(0.8)}{3}} - S = 0.8181 S_{ut} - S$$

where $\mathbf{X} = (S_{ut}, S)$.

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_{\mathbf{X}}) = 0.8181 \mu_{S_{ut}} - \mu_S = 0.8181(220)(10^3) - 100(10^3) = 8.00(10^4) \text{ psi}$$

$$\sigma_Y = \sqrt{\left(\left. \frac{\partial g}{\partial S_{ut}} \right|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_{S_{ut}} \right)^2 + \left(\left. \frac{\partial g}{\partial S} \right|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_S \right)^2}$$

$$= \sqrt{\left(0.8181(20)(10^3)\right)^2 + \left((-1)(10)(10^3)\right)^2}$$

$$= 1.92(10^4) \text{ psi}$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-8.00(10^4)}{1.92(10^4)}\right) = 1.52(10^{-5}) \qquad \textbf{Ans.}$$