16. A shaft undergoes a reversing nomial stress $S_{rev} \sim N(400,\ 2^2)$ MPa with the number of cycles $N_c \sim N(30,000,\ 1000^2)$. The ultimate strength and edurance limit are $S_{ut} = 700$ MPa and $S_e = 280$ MPa, respectively. The fatigure strenghth fraction is determined as $f = 0.85$. If S_{rev} and N_c are independent, estimate the probability of failure using the First Order Second Moment Method.

Solution

The number of cycles to failure is

$$N_f = (\frac{S_{rev}}{a})^{1/b}$$

where

$$a = \frac{(fS_{ut})^2}{S_e} = \frac{[0.85(700)]^2}{280} = 1.264(10^9) \text{ Pa}$$

$$b = -\frac{1}{3}\log\frac{fS_{ut}}{S_e} = -\frac{1}{3}\log\frac{0.85(700)}{300} = -0.1091$$

So the limit-state function is the actual number of cycles subtracted from the number of cycles to failure. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = N_f - N_c = (\frac{S_{rev}}{a})^{1/b} - N_c = (\frac{S_{rev}}{1.264(10^9)})^{1/(-0.1091)} - N_c$$

where $\mathbf{X}$=(S_{rev}, N_c).

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_{\mathbf{X}}) = (\frac{\mu_{S_{rev}}}{1.264(10^9)})^{1/(-0.1091)} - \mu_{N_c} = \left(\frac{400(10^6)}{1.264(10^9)}\right)^{\frac{1}{-0.1091}} - 3(10^4) = 8057$$

$$\sigma_Y = \sqrt{\left(\frac{\partial g}{\partial S_{rev}}\bigg|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_{S_{rev}}\right)^2 + \left(\frac{\partial g}{\partial N_c}\bigg|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_{N_c}\right)^2}$$

$$= \sqrt{\left(\left(\frac{1}{1.264(10^9)}\right)^{\frac{1}{-0.1091}}\left(\frac{1}{-0.1091}\right)\left(\mu_{S_{rev}}\right)^{\frac{1}{-0.1091}-1}\right)\left(\sigma_{S_{rev}}\right)^2 + \left((-1)\sigma_{N_c}\right)^2}$$

$$= \sqrt{\left(\left(\frac{1}{1.264(10^9)}\right)^{\frac{1}{-0.1091}}\left(\frac{1}{-0.1091}\right)\left(400(10^6)\right)^{\frac{1}{-0.1091}-1}\right)\left(2(10^6)\right)^2 + \left((-1)1(10^3)\right)^2}$$

$$= 2010$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-8057}{2010}\right) = 3.06(10^{-5}) \qquad \textbf{Ans.}$$