17. A bending moment $M \sim N(6000,\ 300^2)$ lbf·in is applied to a circular shaft with a diameter of $d \sim N(1, 0.01^2)$ in. If the yield strength is $S_y \sim N(60,\ 6^2)$ kpsi, determine the probability of failure using the First Order Second Moment Method. Note that M, d and S_y are independent.

Solution

The bending stress is given by

$$\sigma = \frac{16M}{\pi d^3}$$

Thus, the limit-state function is the bending stress subtracted from the yield strength. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = S_y - \sigma = S_y - \frac{16M}{\pi d^3}$$

where $\mathbf{X}$=(M, d, S_y).

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_{\mathbf{X}}) = \mu_{S_y} - \frac{16\mu_M}{\pi\mu_d^3} = 60(10^3) - \frac{16(6000)}{\pi(1^3)} = 2.944(10^4)\ \text{psi}$$

$$\sigma_Y = \sqrt{\left(\frac{\partial g}{\partial M}\bigg|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_M\right)^2 + \left(\frac{\partial g}{\partial d}\bigg|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_d\right)^2 + \left(\frac{\partial g}{\partial S_y}\bigg|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_{S_y}\right)^2}$$

$$= \sqrt{\left(-\frac{16}{\pi\mu_d^3}\sigma_M\right)^2 + \left(-(-3)\frac{16\mu_M}{\pi\mu_d^4}\sigma_d\right)^2 + \left(\sigma_{S_y}\right)^2}$$

$$= \sqrt{\left(-\frac{16}{\pi(1^3)}(300)\right)^2 + \left((3)\frac{16(6000)}{\pi(1^4)}(0.01)\right)^2 + \left(6(10^3)\right)^2}$$

$$= 6.259(10^3)\ \text{psi}$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-2.944(10^4)}{6.259(10^3)}\right) = 1.28(10^{-6})$$ **Ans.**