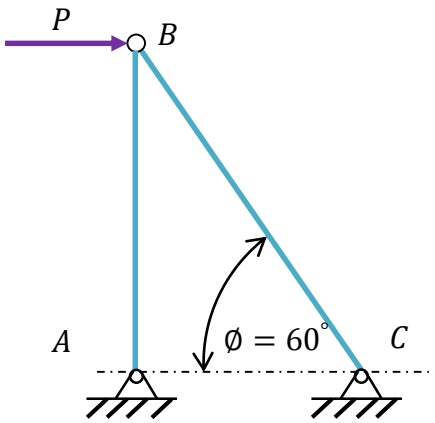
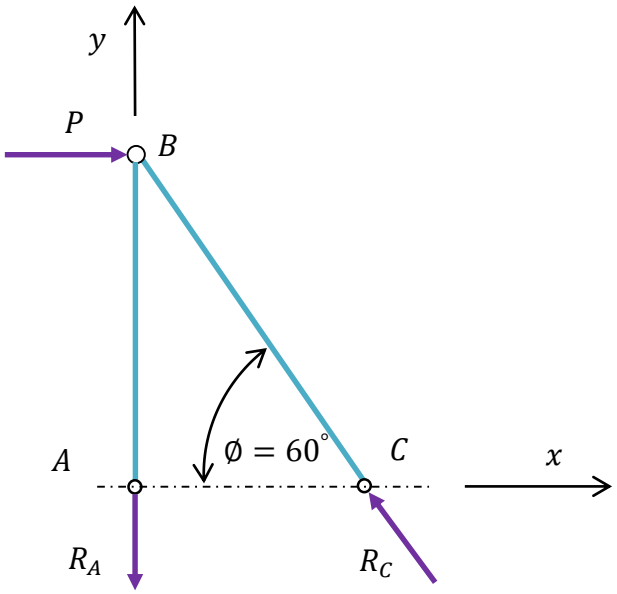
18. A truss undergoes a force $P \sim N(20, 2^2)$ kN. The diamter of rod *BC* is $d = 0.2$ m. If the yield stress of rod *BC* is $S_y \sim N(10,\ 2^2)$ MPa, and *P* and S_y are independent, determine the probability of failure using the First Order Second Moment Method.



Solution

Consider the force equilibrium of rod *AB* and rod *AC* as shown in the figure below



According to the force equilibrium along with x axis,

$$-R_C \cos 60^\circ + P = 0$$

Then

$$R_C = \frac{P}{\cos 60^\circ} = 2P$$

Thus the compressible stress applied to rod *BC* is

$$S = \frac{R_C}{A_{BC}} = \frac{2P}{\frac{\pi d^2}{4}} = \frac{2P}{\frac{\pi (0.2)^2}{4}} = 63.66P$$

The limit-state function is actual stress subtracted from the yield strength. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = S_y - 63.66P = S_y - 63.66P$$

where $\mathbf{X}=(S_y, P)$.

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_\mathbf{X}) = \mu_{S_y} - 63.66\mu_P = 10(10^6) - 63.66(20)(10^3) = 8.727(10^6) \text{ Pa}$$

$$\begin{aligned}
\sigma_Y &= \sqrt{\left(\left.\frac{\partial g}{\partial S_y}\right|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_{S_y}\right)^2 + \left(\left.\frac{\partial g}{\partial P}\right|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_P\right)^2} \\
&= \sqrt{\left((-1)\sigma_{S_y}\right)^2 + (63.66\sigma_P)^2} \\
&= \sqrt{((-1)(2)(10^6))^2 + (63.66(2)(10^3))^2} \\
&= 2.004(10^6) \text{ Pa}
\end{aligned}$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-8.727(10^6)}{2.004(10^6)}\right) = 6.67(10^{-6})$$ **Ans.**