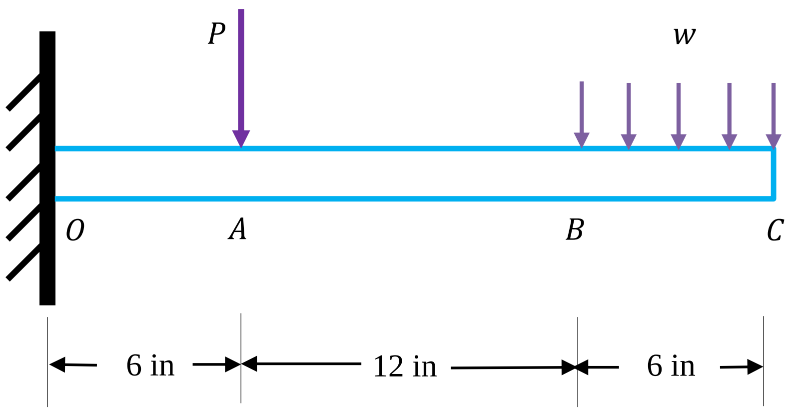
19. As shown in the figure, a beam cantilevered at O is subjected to an uniform load of $w \sim N(100, 10^2)$ lbf/in and a concentrated force $P \sim N(1000,\ 100^2)$ lbf. If the cross-section has a diamater of $d = 2$ in, what is the distirbution of maximum bending stress? Note that w and P are independent.



Solution

Based on the moment equilibrium of beam OC, the bending moment acting at O is

$$M_o = Pl_{OA} + wl_{BC}(l_{OB} + \frac{l_{BC}}{2}) = 6P + 126w$$

Thus the maximum bending stress is given by

$$S = \frac{Mc}{I} = \frac{M_o \frac{d}{2}}{\frac{\pi}{64} d^4} = \frac{32M_o}{\pi d^3} = \frac{32(6P + 126w)}{\pi 2^3} = \frac{24}{\pi} P + \frac{504}{\pi} w$$

Since P and w are independently and normally distributed, their linear sum, S is also normally distributed. And the mean and standard deviation of S are given by

$$\mu_S = \frac{24}{\pi}\mu_P + \frac{504}{\pi}\mu_w = \frac{24}{\pi}(1000) + \frac{504}{\pi}(100) = 1.681(10^5) \ \text{ psi}$$

$$\sigma_S = \sqrt{\left(\frac{24}{\pi}\sigma_P\right)^2 + \left(\frac{504}{\pi}\sigma_w\right)^2} = \sqrt{\left(\frac{24}{\pi}(100)\right)^2 + \left(\frac{504}{\pi}(10)\right)^2} = 1.606(10^4) \ \text{ psi}$$

So the distribution of the maximum bending stress is $S \sim N\left(1.681(10^5),\ \left(1.606(10^4)\right)^2\right)$ psi.