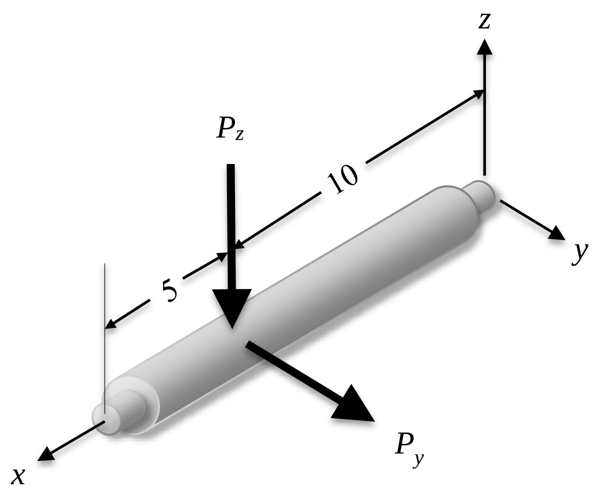
2. A simply-supported steel beam has a diameter of 1.25 in, and design requirements demand that the deflection in any direction at $x = 10$ in should be less than $\delta = 0.00375$ in. Find the probability of failure given that $P_z \sim N(100,8^2)$ lb and $P_y \sim N(200,10^2)$ lb using the First Order Second Moment Method. Note that P_z and P_y are independent.



Solution

Step 1: Define the limit-state function.

$$\delta_y = \frac{-P_y ab}{6EIL}(a^2 + b^2 - L^2)$$

$$\delta_z = \frac{P_z ab}{6EIL}(a^2 + b^2 - L^2)$$

$$\delta = \sqrt{{\delta_y}^2 + {\delta_z}^2} = \sqrt{\left[\frac{-P_y ab}{6EIL}(a^2 + b^2 - L^2)\right]^2 + \left[\frac{P_z ab}{6EIL}(a^2 + b^2 - L^2)\right]^2}$$

Limit-state function:

$$g(\mathbf{X}) = \delta_{max} - \delta \text{ , where } \mathbf{X} = (P_y, P_z).$$

Therefore,

$$g(\mathbf{X}) = \delta_{max} - \sqrt{\left[\frac{-P_y ab}{6EIL}(a^2 + b^2 - L^2)\right]^2 + \left[\frac{P_z ab}{6EIL}(a^2 + b^2 - L^2)\right]^2}$$

Then,

$$g(\mathbf{X}) = \delta_{max} - \frac{-ab}{6EIL}(a^2 + b^2 - L^2)\sqrt{\left({P_y}^2 + {P_z}^2\right)}$$

where a is the distance to forces from origin, L is the length of the beam, $b = L - a$, E is the Young's modulus of steel, and I is the moment of inertia about y or z axis.

Step 2: Differentiate the limit-state function.

$$\left.\frac{\partial g}{\partial P_y}\right|_{\boldsymbol{\mu}_\mathbf{X}} = \frac{-P_y ab(a^2 + b^2 - L^2)}{6EIL\sqrt{\left({P_y}^2 + {P_z}^2\right)}} = \frac{-200 \times 10 \times 5 \times (10^2 + 5^2 - 15^2)}{6 \times 30 \times 10^6 \times \frac{\pi}{64} \times 1.25^4 \times 15 \times \sqrt{200^2 + 100^2}} = -1.38e - 5$$

$$\left.\frac{\partial g}{\partial P_z}\right|_{\boldsymbol{\mu}_\mathbf{X}} = \frac{-P_z ab(a^2 + b^2 - L^2)}{6EIL\sqrt{\left({P_y}^2 + {P_z}^2\right)}} = \frac{-100 \times 10 \times 5 \times (10^2 + 5^2 - 15^2)}{6 \times 30 \times 10^6 \times \frac{\pi}{64} \times 1.25^4 \times 15 \times \sqrt{200^2 + 100^2}} = -6.91e - 6$$

Step 3: Find the mean and standard deviation of the limit-state function.

$$\mu_g = g(\boldsymbol{\mu}_\mathbf{X}) = \delta_{max} - \frac{-ab}{6EIL}(a^2 + b^2 - L^2)\sqrt{\left({\mu_{P_y}}^2 + {\mu_{P_z}}^2\right)} = 0.00375 - 0.00346 = 2.94e - 4 \text{ in}$$

$$\sigma_g = \sqrt{\left(\left.\frac{\partial g}{\partial P_y}\right|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_{P_y}\right)^2 + \left(\left.\frac{\partial g}{\partial P_z}\right|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_{P_z}\right)^2} = 1.49e - 4 \text{ in}$$

Step 4: Evaluate the probability of failure.

$$p_f = \Phi\left(\frac{-\mu_g}{\sigma_g}\right) = 0.0238525$$

Probability of failure: 0.0238525 **Ans.**