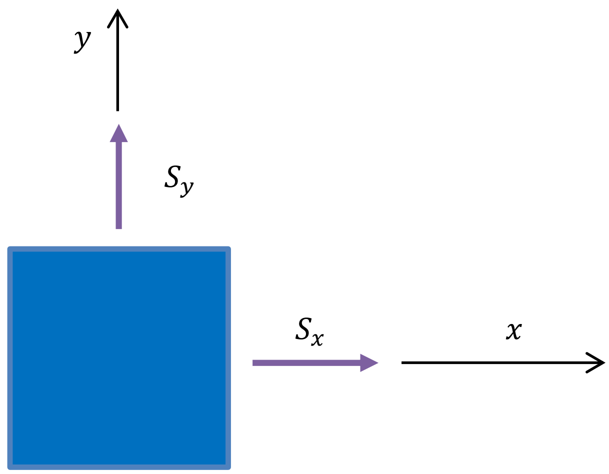
20. A stress element shown in the figure is subjected to two-dimensional stresses S_x and S_y. The axial length of the element is $l_x = 1$ cm. The Poission's ratio is $\nu = 0.3$ and the modulus of elasticity is $E = 30$ MPa. If $S_x \sim N(20,\ 2^2)$ MPa and $S_y \sim N(10, 1^2)$ MPa, estimate the mean and standarad deviation of the axial elongation using the First Order Second Moment Method. Note that S_x and S_y are independent.



Solution

The axial strain is given by

$$\epsilon_x = \frac{1}{E}(S_x - \nu S_y)$$

Thus, the axial elongation is

$$\delta_x = \epsilon_x l_x = \frac{1}{E}(S_x - \nu S_y) l_x$$

Let

$$g(\mathbf{X}) = \delta_x = \frac{1}{E}(S_x - \nu S_y) l_x$$

where **X**=(S_x, S_y).

Using FOSM, we have

$$\mu_{\delta_x} = g(\boldsymbol{\mu}_{\mathbf{X}}) = \frac{1}{E}\left(\mu_{S_x} - \nu \mu_{S_y}\right) l_x = \frac{1}{30(10^6)}\left(20(10^6) - 0.3(10)(10^6)\right)(1)(10^{-2})$$

$$= 5.667(10^{-3})\ \text{m}$$

$$\sigma_{\delta_x} = \sqrt{\left(\frac{\partial g}{\partial S_x}\bigg|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_{S_x}\right)^2 + \left(\frac{\partial g}{\partial S_y}\bigg|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_{S_y}\right)^2}$$

$$= \sqrt{\left(\frac{\mu_{l_x}}{E}\sigma_{S_x}\right)^2 + \left(\frac{-\nu\mu_{l_x}}{E}\sigma_{S_y}\right)^2}$$

$$= \sqrt{\left(\frac{1(10^{-2})}{30(10^6)}2(10^6)\right)^2 + \left(\frac{-0.3(1)(10^{-2})}{30(10^6)}1(10^6)\right)^2}$$

$$= 6.741(10^{-4})\ \text{m}$$