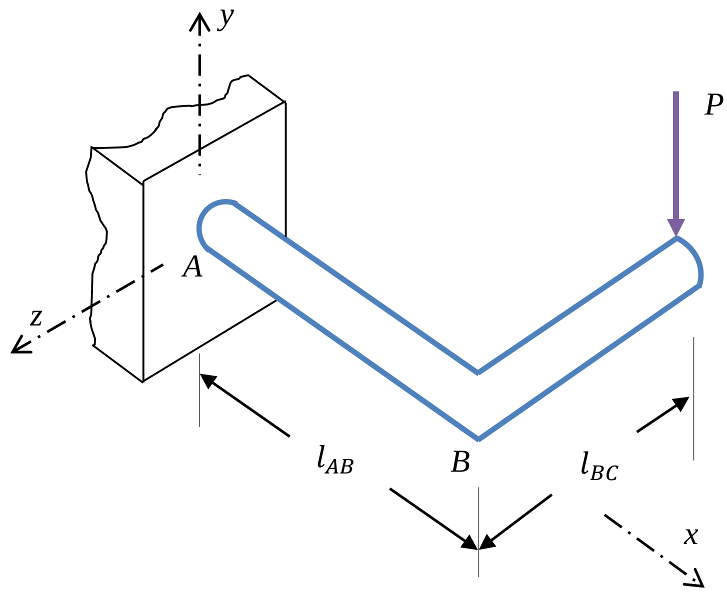
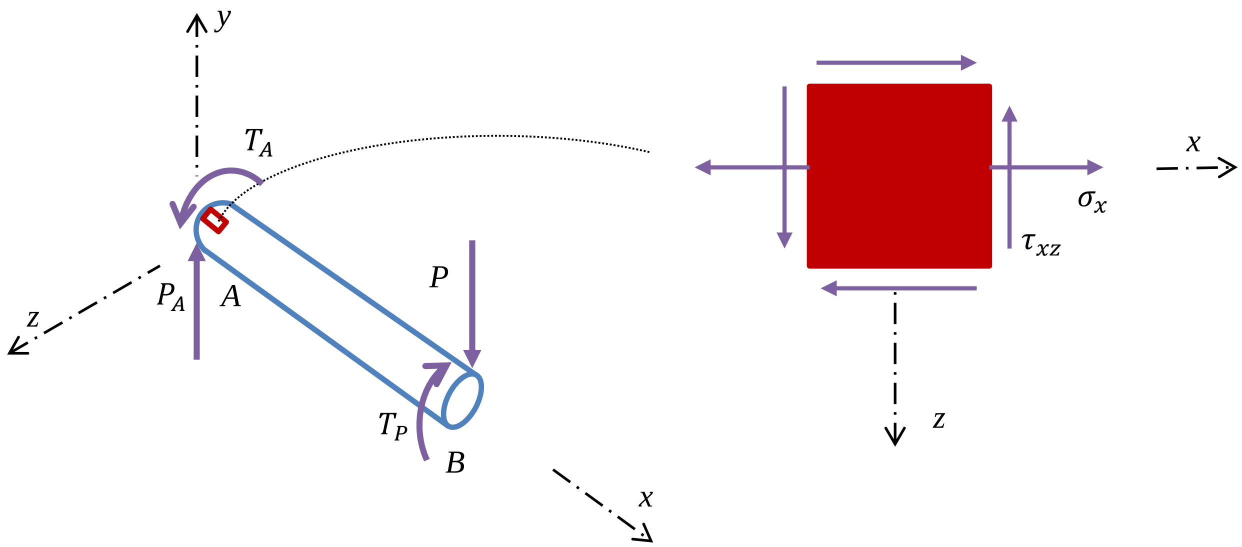
22. A crank shown in the figure is subjected to a force $P \sim N(700, 70^2)$ lbf. The shaft AB is fixed at A and has a diameter of $d = 1$ in. The length of the shaft AB is $l_{AB} = 5$ in, and the length of the arm BC is $l_{BC} = 4$ in. The yield strenght of the shaft AB is $S_y \sim N(80, 8^2)$ kpsi. If P and S_y are independent, estimate the probability of failure using the First Order Second Moment Method. Use the distortion-energy theroy.



Solution

Draw a free-body diagram of the shaft AB and the critical stress element appearing on the top surface at point A.



For the critical stress element, the normal stress resulted from the bending moment is found to be

$$\sigma_x = \frac{Mc}{I} = \frac{Pl_{AB}\frac{d}{2}}{\frac{\pi}{64}d^4} = \frac{32Pl_{AB}}{\pi d^3}$$

And the shear stress resulted from torsion is given by

$$\tau_{xz} = \frac{T_P r}{J} = \frac{Pl_{BC}\frac{d}{2}}{\frac{\pi d^4}{32}} = \frac{16Pl_{BC}}{\pi d^3}$$

where T_P is the torsion resulted from the force P, r is the radius of the circular cross-section, and J is the polar second moment of inertial.

Based on the distortion-energy theory, the von Mises stress is

$$\sigma' = (\sigma_x^2 + 3\tau_{xz}^2)^{1/2}$$

So the limit-state function is the von Mises stress subtracted from the yield strength. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = S_y - \sigma' = S_y - (\sigma_x^2 + 3\tau_{xz}^2)^{1/2} = S_y - \left(\left(\frac{32Pl_{AB}}{\pi d^3}\right)^2 + 3\left(\frac{16Pl_{BC}}{\pi d^3}\right)^2\right)^{1/2}$$

$$= S_y - \frac{16P}{\pi d^3}(4l_{AB}^2 + 3l_{BC}^2)^{\frac{1}{2}} = S_y - 61.9585P$$

where $\mathbf{X}$=(S_y, P).

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_\mathbf{X}) = \mu_{S_y} - 61.9585\mu_P = 80(10^3) - 61.9585(700) = 3.6629(10^4) \text{ psi}$$

$$\sigma_Y = \sqrt{\left(\left.\frac{\partial g}{\partial S_y}\right|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_{S_y}\right)^2 + \left(\left.\frac{\partial g}{\partial P}\right|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_P\right)^2} = \sqrt{\left(\sigma_{S_y}\right)^2 + (-61.9585\sigma_P)^2}$$

$$= \sqrt{\left(8(10^3)\right)^2 + (-61.9585(70))^2}$$

$$= 9.1000(10^3) \text{ psi}$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-3.6629(10^4)}{9.1000(10^3)}\right) = 2.85(10^{-5})$$ **Ans.**