23. A tension rod is subjected to a load of $P \sim N(15,\ 1^2)$ kip. It has a diameter of $d \sim N(1, 0.01^2)$ in and a length of $l \sim N(5, 0.05^2)$ ft. The Poission ratio is $\nu = 0.29$ and the modulus of elasticity is $E = 30(10^6)$ psi. Determine the mean and standard deviation of the change in rod diameter using the First Order Second Moment Method. Note that P, d and l are independent.

Solution

The applied tensile stress is

$$S = \frac{P}{A} = \frac{P}{\frac{\pi d^2}{4}} = \frac{4P}{\pi d^2}$$

And the corresponding axial strain is given by

$$\epsilon_a = \frac{S}{E} = \frac{4P}{\pi d^2 E}$$

Then the lateral strain is

$$\epsilon_l = -\nu \epsilon_a = -\frac{4P\nu}{\pi d^2 E}$$

Thus the change in rod diameter is expressed as

$$\Delta d = \epsilon_l d = -\frac{4\nu}{\pi E}\frac{P}{d}$$

Let

$$Y = g(\mathbf{X}) = \Delta d = -\frac{4\nu}{\pi E}\frac{P}{d}$$

where $\mathbf{X}$=(P, d).

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_\mathbf{X}) = -\frac{4\nu}{\pi E}\frac{\mu_P}{\mu_d} = -\frac{4(0.29)}{\pi 30(10^6)}\frac{15(10^3)}{1} = -1.8462(10^{-4})\ \text{in}$$

$$\sigma_Y = \sqrt{\left(\frac{\partial g}{\partial P}\bigg|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_P\right)^2 + \left(\frac{\partial g}{\partial d}\bigg|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_d\right)^2}$$

$$= \sqrt{\left(-\frac{4\nu}{\pi E}\frac{1}{\mu_d}\sigma_P\right)^2 + \left(\frac{4\nu}{\pi E}\frac{\mu_P}{\mu_d^2}\sigma_d\right)^2}$$

$$= \sqrt{\left(-\frac{4(0.29)}{\pi 30(10^6)}\frac{1}{1}1(10^3)\right)^2 + \left(\frac{4(0.29)}{\pi 30(10^6)}\frac{15(10^3)}{1^2}(0.01)\right)^2}$$

$$= 1.2446(10^{-5})\ \text{in}$$

Ans.