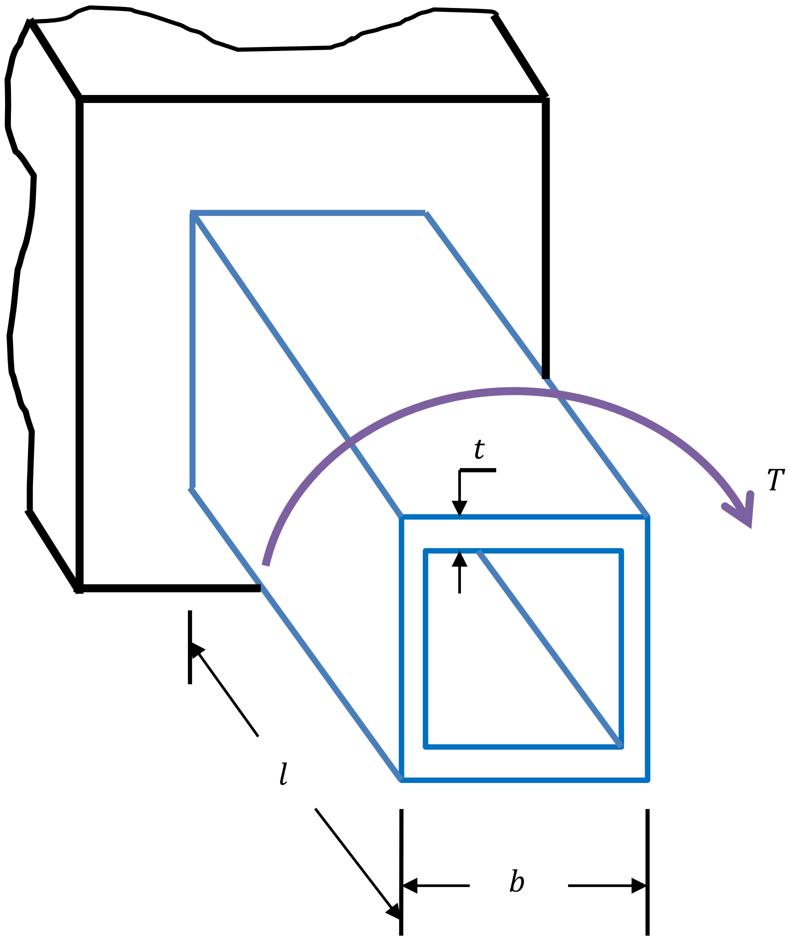
24. A torque $T \sim N(10,000\ \ 1,000^2)$ lbf·in is applied to a steel thin-wall tube. The tube is 40 in long and has a square cross section with side length $b = 2.5$ in and wall thickness $t = 0.25$ in. The shear modulus of the tube is $G = 11.5(10^6)$ psi. If the allowable shear stress is $\tau_a \sim N(12\ \ 2^2)$ kpsi, determine the probability of failure using the First Order Second Moment Method. Note that T and τ_a are independent.



Solution

According to the theory of closed thin-walled tube, the shear stress is

$$\tau = \frac{T}{2A_m t}$$

where A_m is the area enclosed by section median line, given by

$$A_m = (b - t)(b - t) = 5.0625 \ \text{in}^2$$

The limit-state function is the actual shear stress subtracted from the allowable stress. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = \tau_a - \tau = \tau_a - \frac{T}{2A_m t} = \tau_a - 0.3951T$$

where $\mathbf{X}=(\tau_a, T)$.

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_\mathbf{X}) = \mu_{\tau_a} - 0.3951\mu_T = 12(10^3) - 0.3951(10)(10^3) = 8.0494(10^3)\ \text{psi}$$

$$\sigma_Y = \sqrt{\left(\left.\frac{\partial g}{\partial \tau_a}\right|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_{\tau_a}\right)^2 + \left(\left.\frac{\partial g}{\partial T}\right|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_T\right)^2}$$

$$= \sqrt{\left(\sigma_{\tau_a}\right)^2 + (-0.3951\sigma_T)^2}$$

$$= \sqrt{\left(2(10^3)\right)^2 + (-0.3951(1)(10^3))^2}$$

$$= 2.0386(10^3)\ \text{psi}$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-8.0494(10^3)}{2.0386(10^3)}\right) = 3.93(10^{-5})$$ **Ans.**