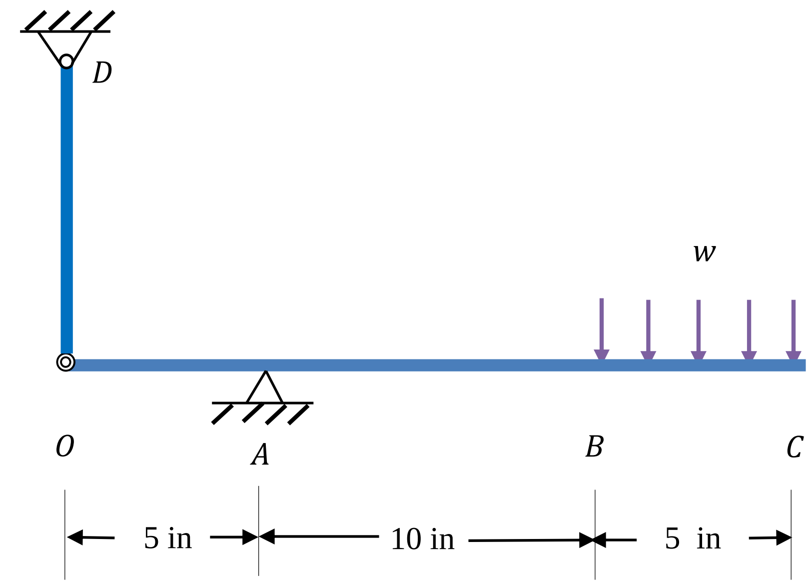
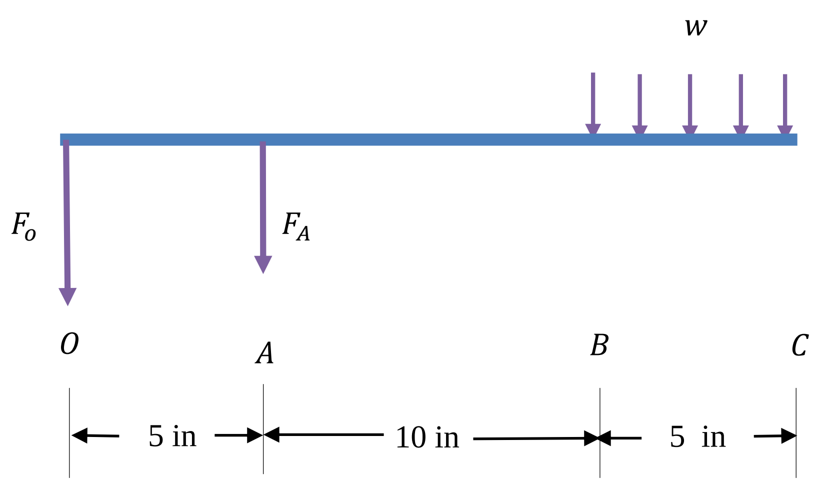
25. An uniform load of $w \sim N(100,\ 10^2)$ lbf/in is applied to a rod *OC* shown in the figure. If the rod *OD* has a diameter of $d = 1$ in, and its yield strength is $S_y \sim N(20, 2^2)$ kips, determine the probability of failure using the First Order Second Moment Method. Note that w and S_y are independent.



Solution

Consider the free body diagram of rod *OC* shown in the figure.



Based on the moment equilibrium of rod *OC* with respect to point *A*,

$$F_o l_{OA} - w l_{BC}(l_{AB} + \frac{l_{BC}}{2}) = 0$$

Solving for F_0 yields

$$F_o = \frac{w l_{BC}(2l_{AB} + l_{BC})}{2l_{OA}}$$

Thus the compressed stress acting on rod *OD* is given by

$$S = \frac{F_o}{A} = \frac{\frac{w l_{BC}(2l_{AB} + l_{BC})}{2l_{OA}}}{\frac{\pi}{4}d^2} = \frac{2l_{BC}(2l_{AB} + l_{BC})}{\pi l_{OA} d^2} w$$

The limit-state function is the actual compressed stress subtracted from the yield strength. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = S_y - S = S_y - \frac{2l_{BC}(2l_{AB} + l_{BC})}{\pi l_{OA} d^2} w = S_y - w$$

where $\mathbf{X}=(S_y, w)$.

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_\mathbf{X}) = \mu_{S_y} - 0.3951\mu_w = 12(10^3) - 0.3951(10)(10^3) = 8.0494(10^3)\ \text{psi}$$

$$\sigma_Y = \sqrt{\left(\frac{\partial g}{\partial S_y}\bigg|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_{S_y}\right)^2 + \left(\frac{\partial g}{\partial w}\bigg|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_w\right)^2}$$

$$= \sqrt{\left(\sigma_{S_y}\right)^2 + (-0.3951\sigma_w)^2}$$

$$= \sqrt{\left(2(10^3)\right)^2 + (-0.3951(1)(10^3))^2}$$

$$= 2.0386(10^3)\ \text{psi}$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-8.0494(10^3)}{2.0386(10^3)}\right) = 3.93(10^{-5})$$ **Ans.**