26. A tension rod is 2 m long with a yield strength of $S_y \sim N(80,\ 1^2)$ MPa. The modulus of elasticity is $E = 120$ GPa. If the axial elongation is measured to be $\delta_a \sim N(1,\ 0.01^2)$ mm, determine the probability of failure using the First Order Second Moment Method. Note that S_y and δ_a are independent.

Solution

According to Hooke's law, the tensile stress is

$$S = E\epsilon = E\frac{\delta_a}{l}$$

where ϵ is the stain and l is the length of tension rod.

The limit-state function is the actual tensile stress subtracted from the yield strength. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = S_y - S = S_y - \frac{E}{l}\delta_a = S_y - \frac{E}{l}\delta_a$$

where **X**=(S_y, δ_a).

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_\mathbf{X}) = \mu_{S_y} - 0.3951\mu_{\delta_a} = 12(10^3) - 0.3951(10)(10^3) = 8.0494(10^3)\ \text{psi}$$

$$\sigma_Y = \sqrt{\left(\left.\frac{\partial g}{\partial S_y}\right|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_{S_y}\right)^2 + \left(\left.\frac{\partial g}{\partial \delta_a}\right|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_{\delta_a}\right)^2}$$

$$= \sqrt{\left(\sigma_{S_y}\right)^2 + \left(-0.3951\sigma_{\delta_a}\right)^2}$$

$$= \sqrt{\left(2(10^3)\right)^2 + (-0.3951(1)(10^3))^2}$$

$$= 2.0386(10^3)\ \text{psi}$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-8.0494(10^3)}{2.0386(10^3)}\right) = 3.93(10^{-5})$$ **Ans.**