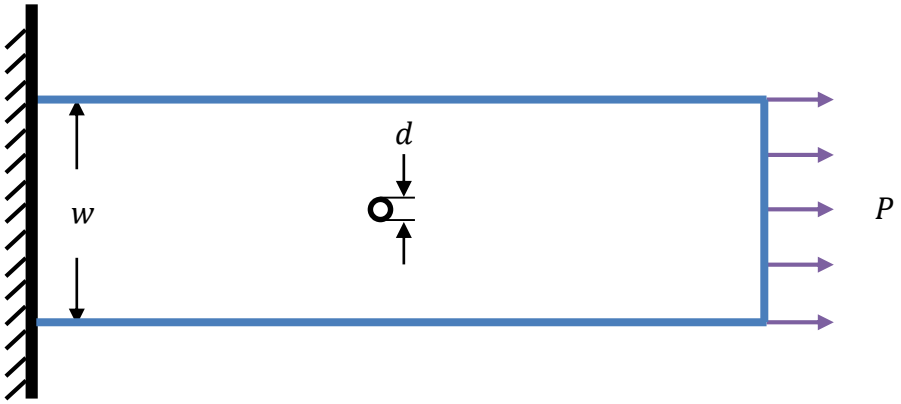
27. A random force $P \sim N(15,\ 2^2)$ KN is applied to a brittle thin plate with a 8 mm-diameter hole in the center. The plate has a width of $w = 80$ mm and a thickness of $t = 2$ mm. If the stress concentration factor is $K_t = 2.7$ and the yield strength of the plate is $S_y \sim N(400, 4^2)$ MPa, estimate the probability of failure using the First Order Second Moment Method. Note that P and S_y are independent.



Solution

The nominal stress is

$$S_n = \frac{P}{A} = \frac{P}{(w-d)t}$$

Considering the effect of stress concentrations, the maximum stress is given by

$$S_{\max} = K_t S_n = \frac{K_t}{(w-d)t} P$$

The limit-state function is the maximum stress subtracted from the yield strength. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = S_y - S_{\max} = S_y - \frac{K_t}{(w-d)t} P = S_y - 1.8750(10^4)P$$

where $\mathbf{X}=(S_y, P)$.

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_\mathbf{X}) = \mu_{S_y} - 1.8750(10^4)\mu_P = 400(10^6) - 1.8750(10^4)(15)(10^3) = 1.1875(10^8) \text{ Pa}$$

$$\sigma_Y = \sqrt{\left(\frac{\partial g}{\partial S_y}\bigg|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_{S_y}\right)^2 + \left(\frac{\partial g}{\partial P}\bigg|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_P\right)^2}$$

$$= \sqrt{\left(\sigma_{S_y}\right)^2 + (-1.8750(10^4)\sigma_P)^2}$$

$$= \sqrt{\left(4(10^6)\right)^2 + (-1.8750(10^4)(2)(10^3))^2}$$

$$= 3.7713(10^7) \text{ Pa}$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-1.1875(10^8)}{3.7713(10^7)}\right) = 8.20(10^{-4})$$ **Ans.**