28. A stress element is subjected to two normal stresses of $S_x \sim N(50,\ 5^2)$ MPa and $S_y \sim N(50,\ 5^2)$ MPa, and a shear stress of $\tau_{xy} \sim N(30,\ 3^2)$ MPa. If S_x, S_y and τ_{xy} are independent, what is the mean and standard deviation of the first principal stress based on the First Order Second Moment Method?

Solution

According to the formula of the first principal stress,

$$S = \frac{S_x + S_y}{2} + \sqrt{\left(\frac{S_x - S_y}{2}\right)^2 + \tau_{xy}^2}$$

Let

$$g(\mathbf{X}) = S = \frac{S_x + S_y}{2} + \sqrt{\left(\frac{S_x - S_y}{2}\right)^2 + \tau_{xy}^2}$$

where $\mathbf{X}=(S_x, S_y, \tau_{xy})$.

Using FOSM, we have

$$\mu_S = g(\boldsymbol{\mu}_{\mathbf{X}}) = \frac{\mu_{S_x} + \mu_{S_y}}{2} + \sqrt{\left(\frac{\mu_{S_x} - \mu_{S_y}}{2}\right)^2 + \mu_{\tau_{xy}}^2}$$

$$= \frac{50(10^6) + 50(10^6)}{2} + \sqrt{\left(\frac{50(10^6) - 50(10^6)}{2}\right)^2 + \left(30(10^6)\right)^2}$$

$$= 80 \text{ MPa}$$

$$\sigma_S = \sqrt{\left(\frac{\partial S}{\partial S_x}\right)^2 \sigma_{S_x}^2 + \left(\frac{\partial S}{\partial S_y}\right)^2 \sigma_{S_y}^2 + \left(\frac{\partial S}{\partial \tau_{xy}}\right)^2 \sigma_{\tau_{xy}}^2}$$

$$= \sqrt{\left(\frac{1}{2} + \frac{\frac{\mu_{S_x} - \mu_{S_y}}{2}}{2\sqrt{\left(\frac{\mu_{S_x} - \mu_{S_y}}{2}\right)^2 + \mu_{\tau_{xy}}^2}}\right)^2 \sigma_{S_x}^2 + \left(\frac{1}{2} - \frac{\frac{\mu_{S_x} - \mu_{S_y}}{2}}{2\sqrt{\left(\frac{\mu_{S_x} - \mu_{S_y}}{2}\right)^2 + \mu_{\tau_{xy}}^2}}\right)^2 \sigma_{S_y}^2 + \left(\frac{\mu_{\tau_{xy}}}{\sqrt{\left(\frac{\mu_{S_x} - \mu_{S_y}}{2}\right)^2 + \mu_{\tau_{xy}}^2}}\right)^2 \sigma_{\tau_{xy}}^2}$$

$$= \sqrt{\left(\frac{1}{2} + \frac{\frac{50(10^6) - 50(10^6)}{2}}{2\sqrt{\left(\frac{50(10^6) - 50(10^6)}{2}\right)^2 + \left(30(10^6)\right)^2}}\right)^2 \left(5(10^6)\right)^2 + \left(\frac{1}{2} - \frac{\frac{50(10^6) - 50(10^6)}{2}}{2\sqrt{\left(\frac{50(10^6) - 50(10^6)}{2}\right)^2 + \left(30(10^6)\right)^2}}\right)^2 \left(5(10^6)\right)^2 + \left(\frac{30(10^6)}{\sqrt{\left(\frac{50(10^6) - 50(10^6)}{2}\right)^2 + \left(30(10^6)\right)^2}}\right)^2 \left(3(10^6)\right)^2}$$

$= 4.64$ MPa

Ans.