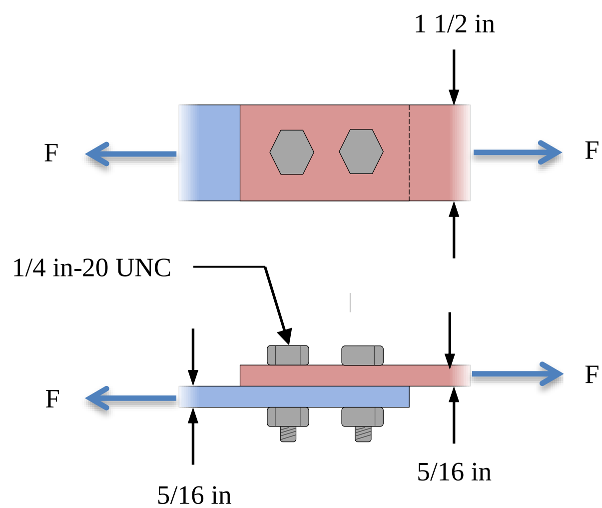
3. For a particular design, engineers decide to use a bolted lap joint to connect two steel cables with flat plate ends. The joint consists of two identical bolts and two identical cold-drawn steel plates. The tension in the joint is distributed with $F \sim N(4, 0.5^2)$ kips. The yield strength of the bolts and the members are distributed normally as well with parameters $B \sim N(100, 4^2)$ ksi and $M \sim N(50, 5^2)$ ksi, respectively. If all the random variables are independent, use MCS to find the probability of failure of the joint based on a) bolt shearing failure, b) bolt bearing failure, c) member bearing failure, and d) member tensile yielding.



Solution

a) Bolt shearing failure

The generic limit-state function is given by

$$Y = g(\mathbf{X}) \tag{1}$$

where $\mathbf{X} = (F, B)$ and failure occurs when $Y < 0$.

The cross-sectional area of a single bolt is

$$A_s = \frac{\pi}{4} D^2 \tag{2}$$

where D is the diameter of the bolt.

With $n = 2$ (bolts),

Single-shear shearing stress:

$$\tau = \frac{F}{nA_s} = \frac{F}{2A_s} \tag{3}$$

Bolt shearing strength:

$$B_s = \frac{B}{\sqrt{3}} \approx .577B \tag{4}$$

Combining Equations (1)-(4), yields

$$g(\mathbf{X}) = B_s - \tau$$

$$g(\mathbf{X}) = .577B - \frac{2F}{\pi D^2} \tag{5}$$

Using MCS with Equation (5) as the limit-state function and 1e7 samples, the probability of failure due to the bolts shearing is 1.2229e-03. **Ans.**

b) Bolt bearing failure

The generic limit-state function is given by

$$Y = g(\mathbf{X}) \tag{6}$$

where $\mathbf{X} = (F, B)$ and failure occurs when $Y < 0$.

Projected area of a single bolt:

$$A_p = tD \tag{7}$$

where *t* is the thickness of the smallest member.

With $n = 2$ (bolts),

Bearing stress across the bolts:

$$\sigma_B = \frac{F}{nA_p} = \frac{F}{2A_p} \tag{8}$$

Combining Equations (6)-(8), yields

$$g(\mathbf{X}) = B - \sigma_B$$

$$g(\mathbf{X}) = B - \frac{F}{2tD} \tag{9}$$

Using MCS with Equation (9) as the limit-state function and 1e7 samples, the probability of failure due to the bolt bearing stress yielding is 0. **Ans.**

c) Member bearing failure

The generic limit-state function is given by

$$Y = g(\mathbf{X}) \tag{10}$$

where $\mathbf{X} = (F, M)$ and failure occurs when $Y < 0$.

Combining Equations (7), (8), and (10), yields

$$g(\mathbf{X}) = M - \sigma_B$$

$$g(\mathbf{X}) = M - \frac{F}{2tD} \tag{11}$$

Using MCS with Equation (11) as the limit-state function and 1e7 samples, the probability of failure due to the member bearing stress yielding is 2.2500e-05. **Ans.**

d) Member tensile yielding

The generic limit-state function is given by

$$Y = g(\mathbf{X}) \tag{12}$$

where $\mathbf{X} = (F, M)$ and failure occurs when $Y < 0$.

Area in tension:

$$A_t = (w - D)t \tag{13}$$

where *w* is the overall width of the members.

Tensional stress in the members:

$$\sigma_t = \frac{F}{A_t} \tag{14}$$

Combining Equations (12)-(14), yields

$$g(\mathbf{X}) = M - \sigma_t$$

$$g(\mathbf{X}) = M - \frac{F}{(w - D)t} \tag{15}$$

Using MCS with Equation (15) as the limit-state function and 1e7 samples, the probability of failure due to the member tensional yielding is 0. **Ans.**

Comparing the probability of failure for each of the modes of failure, it can be seen that the mode most likely to occur first is **bolt shearing**. **Ans.**