30. A cantilever bar is subjected to a torsion $T \sim N(2,\ 0.2^2)$ kN·m at the end. The bar has a round cross section with a diameter of $d \sim N(20,\ 0.2^2)$ mm and a length of $l = 500$ mm. The shear modulus of the bar is $G = 80$ GPa. If T and d are independent, what is the mean and standard deviation of strain energy for torsion using the First Order Second Moment Method?

Solution

The strain energy for torsion is given by

$$U = \frac{T^2 l}{2GJ} = \frac{T^2 l}{2G\frac{\pi d^4}{32}} = \frac{16lT^2}{\pi G d^4}$$

Let

$$Y = g(\mathbf{X}) = U = \frac{16lT^2}{\pi G d^4} = \frac{16(500)(10^{-3})T^2}{\pi(80)(10^9)d^{4^4}} = 3.1831(10^{-11})\frac{T^2}{d^4}$$

where $\mathbf{X}=(T, d)$.

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_\mathbf{X}) = \frac{16l\mu_T^2}{\pi G \mu_d^4} = 3.1831(10^{-11})\frac{\left((2)(10^3)\right)^2}{\left((20)(10^{-3})\right)^4} = 6.3662(10^{-3})\ \text{J}$$

$$\sigma_Y = \sqrt{\left(\left.\frac{\partial g}{\partial T}\right|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_T\right)^2 + \left(\left.\frac{\partial g}{\partial d}\right|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_d\right)^2}$$

$$= \sqrt{\left(3.1831(10^{-11})\frac{2\mu_T}{\mu_d^4}\sigma_T\right)^2 + \left(3.1831(10^{-11})(-4)\frac{\mu_T^2}{\mu_d^5}\sigma_d\right)^2}$$

$$= \sqrt{\begin{array}{l}\left(3.1831(10^{-11})\dfrac{2(2)(10^3)}{\left((20)(10^{-3})\right)^4}(0.2)(10^3)\right)^2 \\ +\left(3.1831(10^{-11})(-4)\dfrac{\left((2)(10^3)\right)^2}{\left((20)(10^{-3})\right)^5}(0.2)(10^{-3})\right)^2\end{array}}$$

$$= 1.2748(10^{-3})\ \text{J}$$

Ans.