31. A torsion-bar with a round cross section is subjected to a torque $T \sim N(3000, 300^2)$ N·m. The diameter of the cross section is $d \sim N(80, 1^2)$ mm and the allowable shear stress is $\tau_a \sim N(50, 5^2)$ MPa, determine the probability of failure using the First Order Second Moment Method. Note that T, d and τ_a are independent.

Solution

The maximum shear stress is

$$\tau_{\max} = \frac{Tr}{J} = \frac{Td}{2J} = \frac{Td}{2\frac{\pi}{32}d^4} = \frac{16T}{\pi d^3}$$

where J is the polar second moment of area.

The limit-state function is the maximum shear stress subtracted from the allowable stress. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = \tau_a - \tau_{\max} = \tau_a - \frac{16T}{\pi d^3}$$

where **X**$=(\tau_a, T, d)$.

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_\mathbf{X}) = \mu_{\tau_a} - \frac{16\mu_T}{\pi\mu_d^3} = 50(10^6) - \frac{16(3000)}{\pi\big((80)(10^{-3})\big)^3} = 2.0158(10^7)\ \text{Pa}$$

$$\sigma_Y = \sqrt{\left(\frac{\partial g}{\partial \tau_a}\bigg|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_{\tau_a}\right)^2 + \left(\frac{\partial g}{\partial T}\bigg|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_T\right)^2 + \left(\frac{\partial g}{\partial d}\bigg|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_d\right)^2}$$

$$= \sqrt{\left(\sigma_{\tau_a}\right)^2 + \left(-\frac{16}{\pi\mu_d^3}\sigma_T\right)^2 + \left(-(-3)\frac{16\mu_M}{\pi\mu_d^4}\sigma_d\right)^2}$$

$$= \sqrt{\big(5(10^6)\big)^2 + \left(-\frac{16}{\pi\big((80)(10^{-3})\big)^3}(300)\right)^2 + \left((3)\frac{16(3000)}{\pi\big((80)(10^{-3})\big)^4}(1)(10^{-3})\right)^2}$$

$$= 5.9294(10^6)\ \text{Pa}$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-2.0158(10^7)}{5.9294(10^6)}\right) = 3.3723(10^{-4})$$ **Ans.**