32. A torsion bar consists of a circular cross secion with a diameter $d \sim N(1,\ 0.01^2)$ in. The bar has a lenghth of $l \sim N(10,\ 0.1^2)$ in and is subjected to a toque $T \sim N(50,\ 5^2)$ lbf·in applied at the central portion of the span. The shear modulus of the bar is $G = 12$ Mpsi. If d, l and T are independent, what is the mean and standard deviation of the spring rate using the First Order Second Moment Method?

Solution

For a torsion bar, the spring rate is

$$k = \frac{GJ}{L}$$

For this problem with two springs in parallel, the total spring rate is given by

$$k_t = \frac{GJ}{\frac{l}{2}} + \frac{GJ}{l - \frac{l}{2}} = \frac{4GJ}{l} = \frac{4G}{l}\frac{\pi}{32}d^4 = \frac{G\pi d^4}{8l}$$

Let

$$Y = g(\mathbf{X}) = k_t = \frac{\pi G d^4}{8l}$$

where $\mathbf{X}$=(d, l).

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_\mathbf{X}) = \frac{\pi G \mu_d^4}{8\mu_l} = \frac{\pi(12)(10^6)(1)^4}{8(10)} = 4.7124(10^5)\ \text{lbf·in/rad}$$

$$\sigma_Y = \sqrt{\left(\left.\frac{\partial g}{\partial d}\right|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_d\right)^2 + \left(\left.\frac{\partial g}{\partial l}\right|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_l\right)^2}$$

$$= \sqrt{\left(\frac{\pi G \mu_d^3}{2\mu_l}\sigma_d\right)^2 + \left(-\frac{\pi G \mu_d^4}{8\mu_l^2}\sigma_l\right)^2}$$

$$= \sqrt{\left(\frac{\pi(12)(10^6)(1)^4}{2(10)}(0.01)\right)^2 + \left(-\frac{\pi(12)(10^6)(1)^4}{8(10)^2}(0.1)\right)^2}$$

$$= 1.9430(10^4)\ \text{lbf·in/rad}$$

Ans.