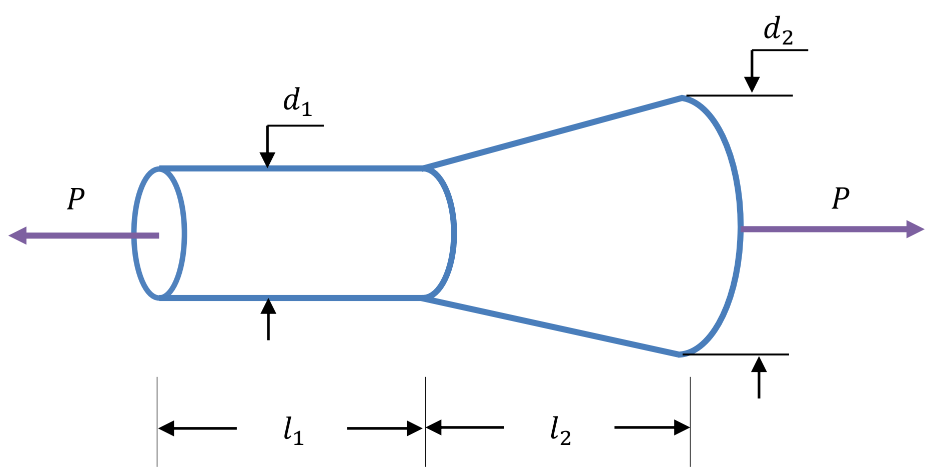
33. A bar is subjected to a tensional force $P \sim N(1000, 100^2)$ lbf. The bar includes a round cross section with diameter $d_1 = 1$ in and length $l_1 \sim N(3, 0.03^2)$ in. And it has a tapered portion of length $l_2 \sim N(3, 0.03^2)$ in and a diameter $d_2 = 2$ in of the end circular cross section. The modulus of elasticity is $E = 30$ Mpsi. If P and l are independent, determine the mean and standard deviation of total axial elongation using the First Order Second Moment Method. Note that the elongation of tapered portion is $\delta = \frac{4}{\pi}\frac{Pl}{d_1 d_2 E}$.



Solution

For the section with constant diameter, the axial elongation is

$$\delta_1 = \frac{Pl_1}{A_1 E} = \frac{Pl_1}{\frac{\pi d_1^2}{4} E} = \frac{4Pl_1}{\pi d_1^2 E}$$

For the tapered portion, the elongation is given by

$$\delta_2 = \frac{4}{\pi}\frac{Pl_2}{d_1 d_2 E}$$

Thus the total axial elongation is expressed as

$$\delta_t = \delta_1 + \delta_2 = \frac{4Pl_1}{\pi d_1^2 E} + \frac{4}{\pi}\frac{Pl_2}{d_1 d_2 E} = 4.2441(10^{-8})Pl_1 + 2.1221(10^{-8})Pl_2$$

Let

$$g(\mathbf{X}) = \delta_t = 4.2441(10^{-8})Pl_1 + 2.1221(10^{-8})Pl_2$$

where **X**=(P, l_1, l_2).

Using FOSM, we have

$$\mu_{\delta_t} = g(\boldsymbol{\mu}_{\mathbf{X}}) = 4.2441(10^{-8})\mu_P\mu_{l_1} + 2.1221(10^{-8})\mu_P\mu_{l_2}$$

$$= 4.2441(10^{-8})(1000)(3) + 2.1221(10^{-8})(1000)(3)$$

$$= 1.9099(10^{-4}) \text{ in}$$

$$\delta_{\delta_t} = \sqrt{\left(\frac{\partial g}{\partial P}\right)^2 \sigma_P^2 + \left(\frac{\partial S}{\partial l_1}\right)^2 \sigma_{l_1}^2 + \left(\frac{\partial S}{\partial l_2}\right)^2 \sigma_{l_2}^2}$$

$$= \sqrt{\begin{array}{c}\left(4.2441(10^{-8})\mu_{l_1} + 2.1221(10^{-8})\mu_{l_2}\right)^2\sigma_P^2 \\ +(4.2441(10^{-8})\mu_P)^2\sigma_{l_1}^2 + (2.1221(10^{-8})\mu_P)^2\sigma_{l_2}^2\end{array}}$$

$$= \sqrt{\begin{array}{c}\left(4.2441(10^{-8})(3) + 2.1221(10^{-8})(3)\right)^2 100^2 \\ +\left(4.2441(10^{-8})(1000)\right)^2(0.03)^2 + \left(2.1221(10^{-8})(1000)\right)^2(0.03)^2\end{array}}$$

$$= 1.9152(10^{-5}) \text{ in}$$

Ans.