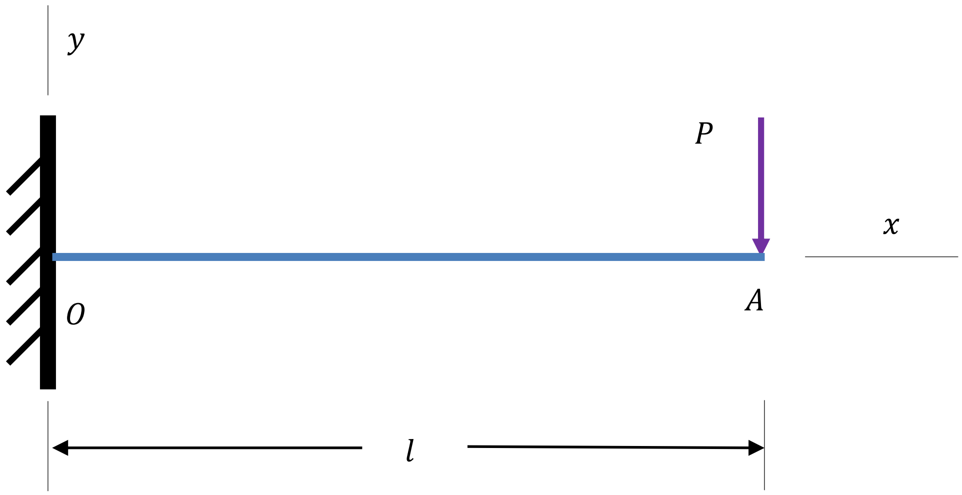
34. A tube is subejcted to a load $P \sim N(700,\ 70^2)$ lbf, shown in the figure. It has an outside diameter of $d_O = 2$ in and insider diameter of $d_i = 1$ in. The lenghth of the tube is $l \sim N(50,\ 0.1^2)$ in and the modulus of elasticity is $E = 11$ Mpsi. The allowable transverse deflection is $y_a = 5$ in. If P and l are independent, determine the probability of failure using the First Order Second Moment Method. Note that the maximum deflection is $y_{\max} = \frac{Fl^3}{3EI}$



Solution

The maximum deflection happens at A, given by

$$y_{\max} = \frac{Pl^3}{3EI} = \frac{Pl^3}{3E\frac{\pi}{64}(d_o^4 - d_i^4)} = \frac{64Pl^3}{3\pi E(d_o^4 - d_i^4)} = 4.1155(10^{-8})Pl^3$$

The limit-state function is the maximum deflection subtracted from allowable transverse deflection. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = y_a - y_{\max} = y_a - \frac{64Pl^3}{3\pi E(d_o^4 - d_i^4)} = 5 - 4.1155(10^{-8})Pl^3$$

where $\mathbf{X}$=(P, l).

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_{\mathbf{X}}) = 5 - 4.1155(10^{-8})\mu_P\mu_l^3 = 5 - 4.1155(10^{-8})(700)(50^3) = 1.3989 \text{ in}$$

$$\sigma_Y = \sqrt{\left(\frac{\partial g}{\partial P}\bigg|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_P\right)^2 + \left(\frac{\partial g}{\partial l}\bigg|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_l\right)^2}$$

$$= \sqrt{(-4.1155(10^{-8})\mu_l^3\sigma_P)^2 + (-(4.1155(10^{-8}))(3)(\mu_P\mu_l^2)\sigma_l)^2}$$

$$= \sqrt{(-4.1155(10^{-8})(50^3)(70))^2 + \left(-\left(4.1155(10^{-8})\right)(3)(700)(50)^2(0.1)\right)^2}$$

$$= 3.6076(10^{-1}) \text{ in}$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-1.3989}{3.6076}\right) = 5.2715(10^{-5})$$ **Ans.**