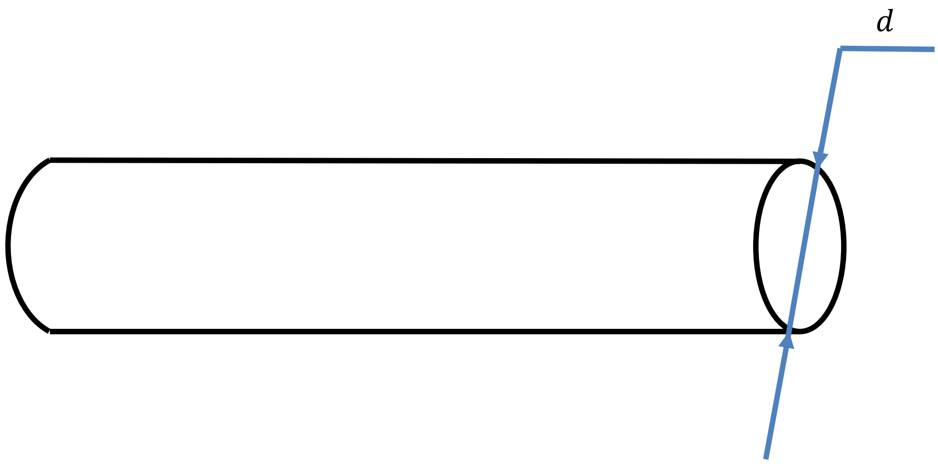
35. A shaft has a circular cross section with a diameter of $d \sim N(50, 0.5^2)$ mm. The allowable shear stress of the shaft is $\tau_a \sim N(100, 10^2)$ MPa. If d and τ_a are independent, estimate the mean and standard deviation of the power that can be transmitted at 3000 rpm using the First Order Second Moment Method.



Solution

The maximum shear stress is given by

$$\tau_{max} = \frac{16T}{\pi d^3}$$

The maximum shear stress should be below the allowable shear stress,

$$\tau_{max} \leq \tau_a$$

Thus the allowable torque is

$$T_a = \frac{\tau_a \pi d^3}{16}$$

And the transmitted power is

$$H = \frac{T_a n}{9.55} = \frac{\tau_a \pi d^3}{16} \frac{n}{9.55} = \frac{\pi n}{16(9.55)} \tau_a d^3 = 61.6805 \tau_a d^3$$

Let

$$g(\mathbf{X}) = H = 61.6805 \tau_a d^3$$

where **X**=(τ_a, d).

Using FOSM, we have

$$\mu_H = g(\boldsymbol{\mu}_{\mathbf{X}}) = 61.6805\mu_{\tau_a}\mu_d^3 = 61.6805(100)(10^6)\left((50)(10^{-3})\right)^3 = 771.0 \text{ kW}$$

$$\sigma_H = \sqrt{\left(\frac{\partial g}{\partial \tau_a}\right)^2 \sigma_{\tau_a}^2 + \left(\frac{\partial g}{\partial d}\right)^2 \sigma_d^2}$$

$$= \sqrt{(61.6805\mu_d^3)^2\sigma_{\tau_a}^2 + \left(61.6805(3)\mu_{\tau_a}\mu_d^2\right)^2\sigma_d^2}$$

$$= \sqrt{\begin{array}{c}\left(61.6805\left((50)(10^{-3})\right)^3\right)^2\left((10)(10^6)\right)^2 + \\ \left(61.6805(3)(100)(10^6)\left((50)(10^{-3})\right)^2\right)^2\left((0.5)(10^{-3})\right)^2\end{array}}$$

$$= 80.5 \text{ kW}$$

Ans.