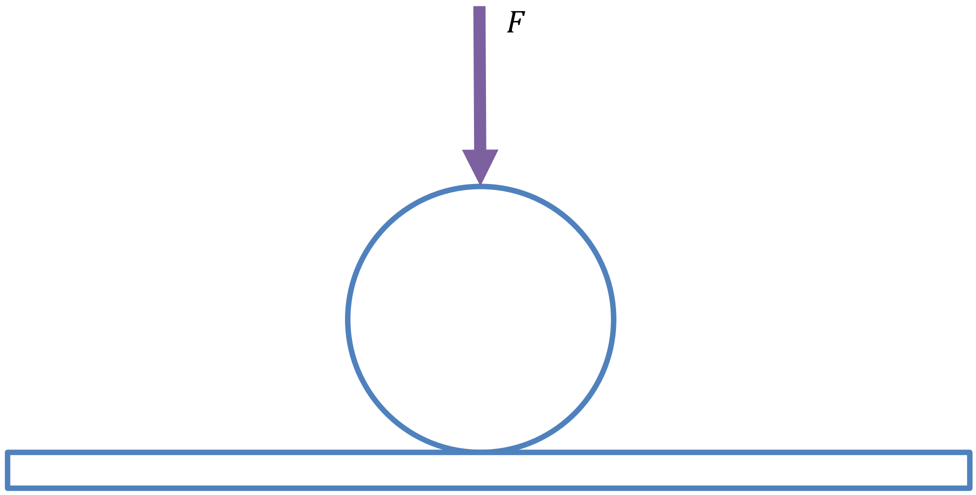
36. A steel ball is placed against a steel plate and is subjected to a force $F \sim N(60,\ 6^2)$ N. The diameter, modulus of elasticity, and Poisson's ratio are $d \sim N(50,\ 0.1^2)$ mm, $E = 207$ GPa and $\upsilon = 0.3$, repectively. If d and F are independent, what is the mean and standard deviation of the maximum pressure that occurs at the contact area?



Solution

The radius of the circular area of contact is given by

$$a = \sqrt[3]{\frac{3F}{8}\frac{\frac{1-\nu_1^2}{E_1}+\frac{1-\nu_2^2}{E_2}}{\frac{1}{d_1}+\frac{1}{d_2}}} = \sqrt[3]{\frac{3F}{8}\frac{2\frac{1-\nu^2}{E}}{\frac{1}{d}+\frac{1}{\infty}}} = \sqrt[3]{\frac{3}{4}\frac{1-\nu^2}{E}Fd}$$

Thus the maximum pressure occurring at contact area is

$$p = \frac{3F}{2\pi a^2} = \frac{3F}{2\pi\left(\frac{3}{4}\frac{1-\nu^2}{E}\right)^{\frac{2}{3}}(Fd)^{\frac{2}{3}}} = \frac{3}{2\pi\left(\frac{3}{4}\frac{1-\nu^2}{E}\right)^{\frac{2}{3}}}F^{\frac{1}{3}}d^{-\frac{2}{3}} = 2.1554(10^7)F^{\frac{1}{3}}d^{-\frac{2}{3}}$$

Let

$$g(\mathbf{X}) = p = 2.1554(10^7)F^{\frac{1}{3}}d^{-\frac{2}{3}}$$

where $\mathbf{X}=(F,d)$.

Using FOSM, we have

$$\mu_p = g(\boldsymbol{\mu}_{\mathbf{X}}) = 2.1554(10^7)\mu_F^{\frac{1}{3}}\mu_d^{-\frac{2}{3}} = 2.1554(10^7)(60)^{\frac{1}{3}}\left(50(10^{-3})\right)^{-\frac{2}{3}} = 6.22(10^8)\ \text{Pa}$$

$$\sigma_p = \sqrt{\left(\frac{\partial g}{\partial F}\right)^2 \sigma_F^2 + \left(\frac{\partial g}{\partial d}\right)^2 \sigma_d^2}$$

$$= \sqrt{\left(2.1554(10^7)(\frac{1}{3})\mu_F^{-\frac{2}{3}}\mu_d^{-\frac{2}{3}}\right)^2 \sigma_F^2 + \left(2.1554(10^7)(-\frac{2}{3})\mu_F^{\frac{1}{3}}\mu_d^{-\frac{5}{3}}\right)^2 \sigma_d^2}$$

$$= \sqrt{\begin{array}{c}\left(2.1554(10^7)(\frac{1}{3})(60)^{-\frac{2}{3}}\left(50(10^{-3})\right)^{-\frac{2}{3}}\right)^2 (6)^2 + \\ \left(2.1554(10^7)(-\frac{2}{3})(60)^{\frac{1}{3}}\left(50(10^{-3})\right)^{-\frac{5}{3}}\right)^2 \left((0.1)(10^{-3})\right)^2\end{array}}$$

$$= 2.07(10^7)\ \text{Pa}$$

Ans.