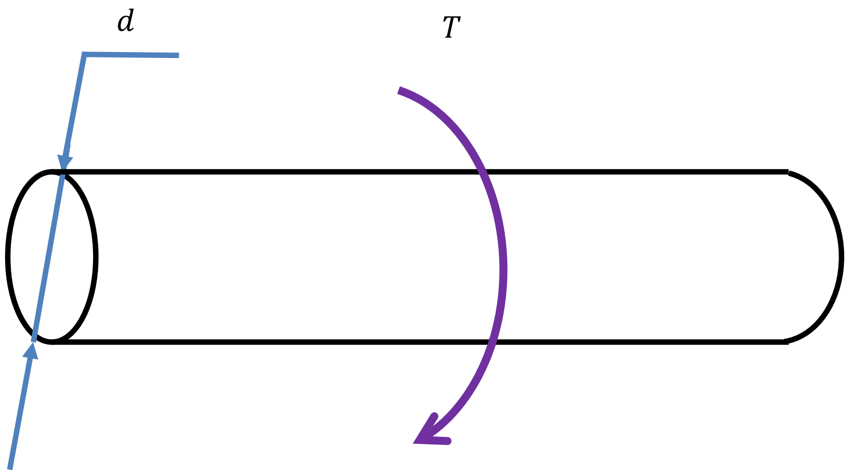
37. A torque $T \sim N(1000, 100^2)$ N·m is applied to a bar with a round cross section. The diameter of the bar is $d \sim N(80, 0.1^2)$ mm. If the allowable shear stress is $\tau_a \sim N(19, 2^2)$ MPa, determine the probability of failrue using the First Order Second Moment Method. Note that T, d and τ_a are independent.



Solution

The maximum shear stress is given by

$$\tau_{max} = \frac{Tr}{J} = \frac{T\frac{d}{2}}{\frac{\pi}{32}d^4} = \frac{16T}{\pi d^3}$$

Thus the limit-state function is the maximum stress subtracted from allowable shear stress. Failure occurs when $Y < 0$,

$$Y = g(\mathbf{X}) = \tau_a - \tau_{max} = \tau_a - \frac{16T}{\pi d^3}$$

where $\mathbf{X}$=(T, d, τ_a).

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_\mathbf{X}) = \mu_{\tau_a} - \frac{16\mu_T}{\pi\mu_d^3} = 19(10^6) - \frac{16(1000)}{\pi\left(80(10^{-3})\right)^3} = 9.05(10^6)\ \text{Pa}$$

$$\sigma_Y = \sqrt{\left(\frac{\partial g}{\partial T}\bigg|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_T\right)^2 + \left(\frac{\partial g}{\partial d}\bigg|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_d\right)^2 + \left(\frac{\partial g}{\partial \tau_a}\bigg|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_{\tau_a}\right)^2}$$

$$= \sqrt{\left(-\frac{16}{\pi\mu_d^3}\sigma_T\right)^2 + \left(-(-3)\frac{16\mu_T}{\pi\mu_d^4}\sigma_d\right)^2 + \left(\sigma_{\tau_a}\right)^2}$$

$$= \sqrt{\left(-\frac{16}{\pi\left(80(10^{-3})\right)^3}(100)\right)^2 + \left((3)\frac{16(1000)}{\pi\left(80(10^{-3})\right)^4}(0.1(10^{-3}))\right)^2 + \left(2(10^6)\right)^2}$$

$$= 2.23(10^6)\ \text{Pa}$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-9.05(10^6)}{2.23(10^6)}\right) = 2.54(10^{-5})$$ **Ans.**