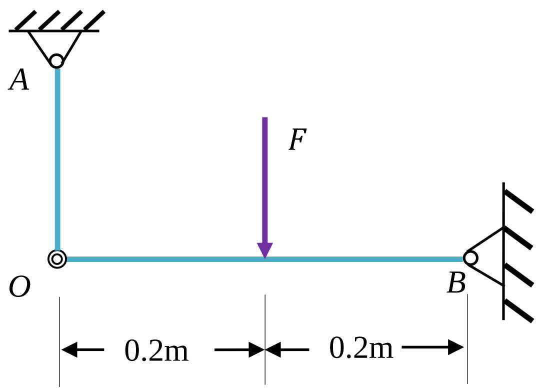
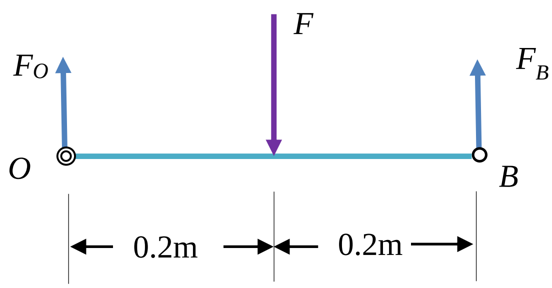
38. Rod *OB* is subjected to a force $F \sim N(1, 0.1^2)$ kN as shown in the figure. The allowable stress of rod *OA* is $S_a \sim N(5, 0.5^2)$ MPa. If rod *OA* has a round cross section with a diameter of $d \sim N(15, 0.1^2)$ mm, determine the probability of failure using the First Order Second Moment Method. Assume that F, d and S_a are independent.



Solution

Consider the free-body diagram of rod *OB*



According to the moment equilibrium of rod *OB* with respect to point *B*,

$$-F_O(0.4) + F(0.2) = 0$$

Solving for F_O yields

$$F_O = \frac{F}{2}$$

Thus the stress acting on the rod *OA* is

$$S = \frac{F_O}{A} = \frac{\frac{F}{2}}{\frac{\pi}{4}d^2} = \frac{2F}{\pi d^2}$$

Thus the limit-state function is the actual stress subtracted from allowable stress. Failure occurs when $Y < 0$,

$$Y = g(\mathbf{X}) = S_a - S = S_a - \frac{2F}{\pi d^2}$$

where $\mathbf{X}$=(F, d, S_a).

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_\mathbf{X}) = \mu_{S_a} - \frac{2\mu_F}{\pi\mu_d^2} = 5(10^6) - \frac{2(1000)}{\pi\left(15(10^{-3})\right)^2} = 2.17(10^6)\ \text{Pa}$$

$$\sigma_Y = \sqrt{\left(\left.\frac{\partial g}{\partial F}\right|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_F\right)^2 + \left(\left.\frac{\partial g}{\partial d}\right|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_d\right)^2 + \left(\left.\frac{\partial g}{\partial S_a}\right|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_{S_a}\right)^2}$$

$$= \sqrt{\left(-\frac{2}{\pi\mu_d^2}\sigma_F\right)^2 + \left(-(-2)\frac{2\mu_F}{\pi\mu_d^3}\sigma_d\right)^2 + \left(\sigma_{S_a}\right)^2}$$

$$= \sqrt{\left(-\frac{2}{\pi\left(15(10^{-3})\right)^2}(100)\right)^2 + \left((2)\frac{2(1000)}{\pi\left(15(10^{-3})\right)^3}(0.1(10^{-3}))\right)^2 + \left(0.5(10^6)\right)^2}$$

$$= 5.76(10^6)\ \text{Pa}$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-2.17(10^6)}{5.76(10^6)}\right) = 8.16(10^{-5})$$ **Ans.**