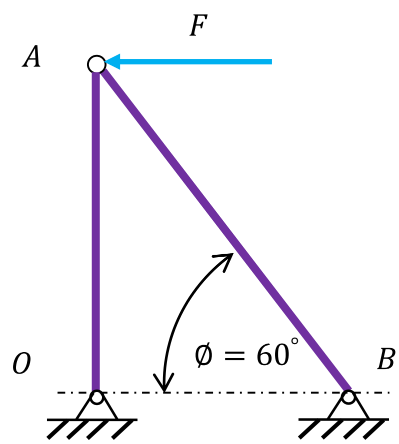
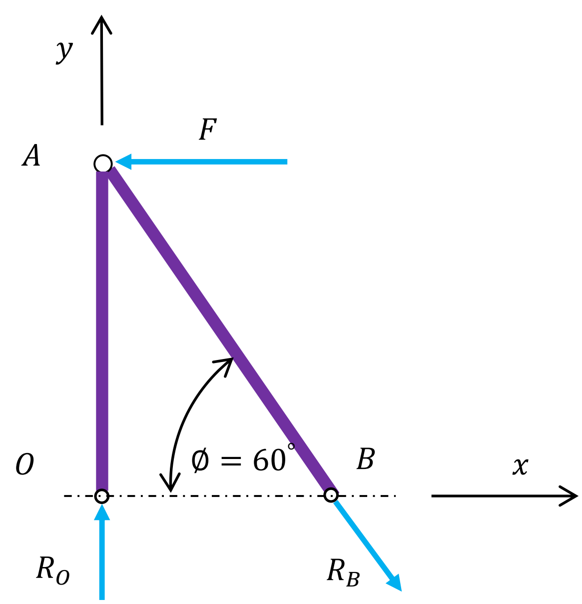
41. A force $F \sim N(8000, 800^2)$ lbf is applied to a truss shown in the figure. The yield strength of rod *AB* is $S_y \sim N(10,\ 1^2)$ kpsi. If the maximum probability of failure is designed to be $p_f = 10^{-5}$, determine the minimum diameter of rod *AB*. Note that F and S_y are independent.



Solution

Consider the force equilibrium of rod *OA* and rod *AB* shown in the figure



According to the force equilibrium along with x axis,

$$+R_B \cos 60^\circ - F = 0$$

Then

$$R_B = \frac{F}{\cos 60^{\circ}} = 2F$$

Thus the tensile stress applied to rod *AB* is

$$S = \frac{R_B}{A_{AB}} = \frac{2F}{\frac{\pi d^2}{4}} = \frac{8}{\pi d^2} F$$

The limit-state function is the actual stress subtracted from the yield strength. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = S_y - \frac{8}{\pi d^2} F$$

where $\mathbf{X}=(S_y, F)$.

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_{\mathbf{X}}) = \mu_{S_y} - \frac{8}{\pi d^2}\mu_F = 10(10^3) - \frac{8}{\pi d^2}(8000)$$

$$\sigma_Y = \sqrt{\left(\frac{\partial g}{\partial S_y}\bigg|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_{S_y}\right)^2 + \left(\frac{\partial g}{\partial P}\bigg|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_F\right)^2}$$

$$= \sqrt{\left(\sigma_{S_y}\right)^2 + \left(-\frac{8}{\pi d^2}\sigma_F\right)^2}$$

$$= \sqrt{((1)(10^3))^2 + \left(-\frac{8}{\pi d^2}(800)\right)^2}$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-\left(10(10^3) - \frac{8}{\pi d^2}(8000)\right)}{\sqrt{((1)(10^3))^2 + \left(-\frac{8}{\pi d^2}(800)\right)^2}}\right) = 10^{-5}$$

Thus

$$\frac{-\mu_Y}{\sigma_Y} = \frac{-\left(10(10^3) - \frac{8}{\pi d^2}(8000)\right)}{\sqrt{((1)(10^3))^2 + \left(-\frac{8}{\pi d^2}(800)\right)^2}} = \Phi^{-1}(10^{-5})$$

Solving for d yields

$$d = 1.98 \text{ in}$$

Thus $d = 2.0$ in can be used. **Ans.**