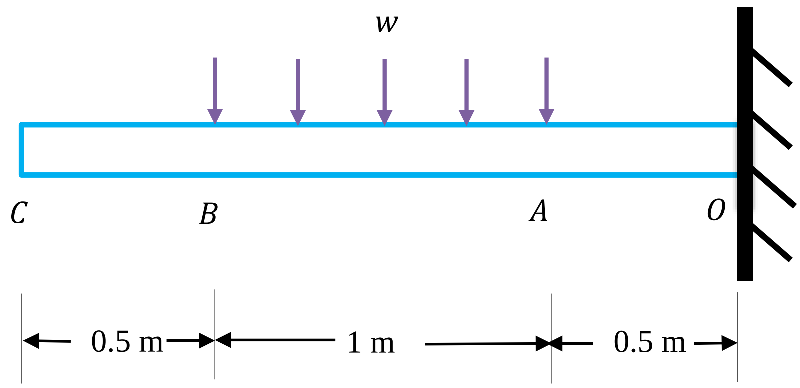
42. A beam with a round cross section is cantilevered at *O* shown in the figure. It is subjected to an uniform load of $w \sim N(900,\ 90^2)$ N/m. If the diameter of the cross section and the allowable bending stress are $d \sim N(60,\ 0.1^2)$ mm and $S_a \sim N(80,\ 8^2)$ MPa, respectively, estimate the probability of failure using the First Order Second Moment Method. Note that w, d and S_a are independent.



Solution

According to the moment equilibrium of beam *OC*, the bending moment acting at *O* is

$$M_o = wl_{AB}\left(l_{OA} + \frac{l_{AB}}{2}\right) = w(1)(0.5 + \frac{1}{2}) = w$$

Thus the maximum bending stress is given by

$$S = \frac{Mc}{I} = \frac{M_o \frac{d}{2}}{\frac{\pi}{64} d^4} = \frac{32M_o}{\pi d^3} = \frac{32w}{\pi d^3}$$

The limit-state function is the maximum bending stress subtracted from the allowable stress. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = S_a - S = S_a - \frac{32w}{\pi d^3}$$

where $\mathbf{X}$=(S_a, w, d).

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_{\mathbf{X}}) = \mu_{S_a} - \frac{32\mu_w}{\pi \mu_d^3} = 80(10^6) - \frac{32(900)}{\pi\left((60)(10^{-3})\right)^3} = 3.76(10^7)\ \text{Pa}$$

$$\sigma_Y = \sqrt{\left(\left.\frac{\partial g}{\partial S_a}\right|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_{S_a}\right)^2 + \left(\left.\frac{\partial g}{\partial w}\right|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_w\right)^2 + \left(\left.\frac{\partial g}{\partial d}\right|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_d\right)^2}$$

$$= \sqrt{\left(\sigma_{S_a}\right)^2 + \left(-\frac{32}{\pi\mu_d^3}\sigma_w\right)^2 + \left(-(-3)\frac{32\mu_w}{\pi\mu_d^4}\sigma_d\right)^2}$$

$$= \sqrt{\left(8(10^6)\right)^2 + \left(-\frac{32}{\pi\left((60)(10^{-3})\right)^3}(90)\right)^2 + \left((3)\frac{32(900)}{\pi\left((60)(10^{-3})\right)^4}(0.1)(10^{-3})\right)^2}$$

$$= 9.06(10^6)\ \text{Pa}$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-3.76(10^7)}{9.06(10^6)}\right) = 1.69(10^{-5})$$ **Ans.**