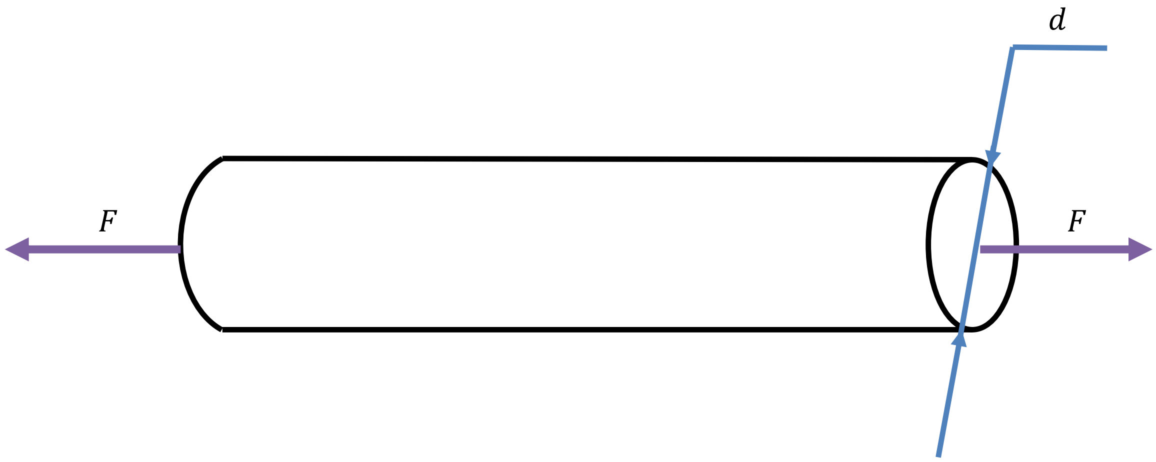
43. A bar with a circular cross section is subjectd to a force $F \sim N(3000, 300^2)$ lbf. The yield strength of the bar is $S_y \sim N(30,\ 3^2)$ kpsi. If the maximum probability of failure is designed to be $p_f = 10^{-5}$, determine the minimum diameter using the First Order Second Moment Method and then select a preferred diameter. Note that F and S_y are independent.



Solution

The tensile stress of the bar is

$$S = \frac{F}{\frac{\pi}{4}d^2} = \frac{4F}{\pi d^2}$$

The limit-state function is the actual tensile stress subtracted from the yield strength. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = S_y - S = S_y - \frac{4F}{\pi d^2}$$

where $\mathbf{X}$=(S_y, F).

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_\mathbf{X}) = \mu_{S_y} - \frac{4}{\pi d^2}\mu_F = 30(10^3) - \frac{4}{\pi d^2}(3000)$$

$$\sigma_Y = \sqrt{\left(\left.\frac{\partial g}{\partial S_y}\right|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_{S_y}\right)^2 + \left(\left.\frac{\partial g}{\partial P}\right|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_F\right)^2}$$

$$= \sqrt{\left(\sigma_{S_y}\right)^2 + \left(-\frac{4}{\pi d^2}\sigma_F\right)^2}$$

$$= \sqrt{((3)(10^3))^2 + \left(-\frac{4}{\pi d^2}(300)\right)^2}$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-\left(30(10^3) - \frac{4}{\pi d^2}(3000)\right)}{\sqrt{((3)(10^3))^2 + \left(-\frac{4}{\pi d^2}(300)\right)^2}}\right) = 10^{-5}$$

Thus

$$\frac{-\mu_Y}{\sigma_Y} = \frac{-\left(30(10^3) - \frac{4}{\pi d^2}(3000)\right)}{\sqrt{((3)(10^3))^2 + \left(-\frac{4}{\pi d^2}(300)\right)^2}} = \Phi^{-1}(10^{-5})$$

Solving for d yields

$$d = 0.495 \text{ in}$$

Thus $d = 0.5$ in can be used. **Ans.**