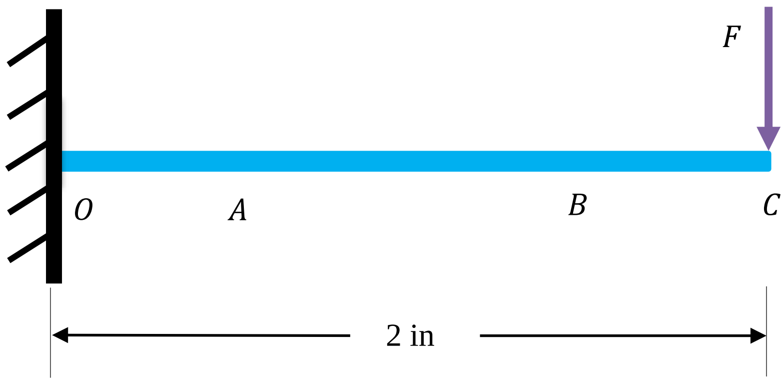
44. A force $F \sim N(1000,\ 100^2)$ lbf is applied to a beam shown in the figure. The beam is cantileverd at point O and has a circular cross section. The allowable bending stress of the beam is $S_a \sim N(80,\ 8^2)$ kpsi. If the maximum probability of failure is designed to be $p_f = 10^{-5}$, determine the minimum diameter of the beam and select a preferred diameter using the First Order Second Moment Method. Note that F and S_a are independent.



Solution

Based on the moment equilibrium of beam OC, the bending moment acting at O is

$$M_o = Fl_{OC} = 2F$$

Thus the maximum bending stress is given by

$$S = \frac{Mc}{I} = \frac{M_o \frac{d}{2}}{\frac{\pi}{64} d^4} = \frac{32M_o}{\pi d^3} = \frac{64F}{\pi d^3}$$

The limit-state function is the maximum bending stress subtracted from the allowable stress. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = S_a - S = S_a - \frac{64F}{\pi d^3}$$

where $\mathbf{X}=(S_a, F)$.

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_\mathbf{X}) = \mu_{S_a} - \frac{64}{\pi d^3}\mu_F = 80(10^3) - \frac{64}{\pi d^3}(1000)$$

$$\sigma_Y = \sqrt{\left(\left.\frac{\partial g}{\partial S_y}\right|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_{S_y}\right)^2 + \left(\left.\frac{\partial g}{\partial P}\right|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_F\right)^2}$$

$$= \sqrt{\left(\sigma_{S_y}\right)^2 + \left(-\frac{64}{\pi d^3}\sigma_F\right)^2}$$

$$= \sqrt{((8)(10^3))^2 + \left(-\frac{64}{\pi d^3}(100)\right)^2}$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-\left(80(10^3) - \frac{64}{\pi d^3}(1000)\ \right)}{\sqrt{((8)(10^3))^2 + \left(-\frac{64}{\pi d^3}(100)\right)^2}}\right) = 10^{-5}$$

Thus

$$\frac{-\mu_Y}{\sigma_Y} = \frac{-\left(80(10^3) - \frac{64}{\pi d^3}(1000)\ \right)}{\sqrt{((8)(10^3))^2 + \left(-\frac{64}{\pi d^3}(100)\right)^2}} = \Phi^{-1}(10^{-5})$$

Solving for d yields

$$d = 0.789 \text{ in}$$

Thus $d = 0.80$ in can be used. **Ans.**