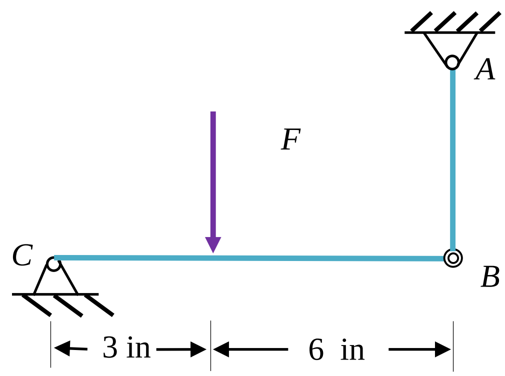
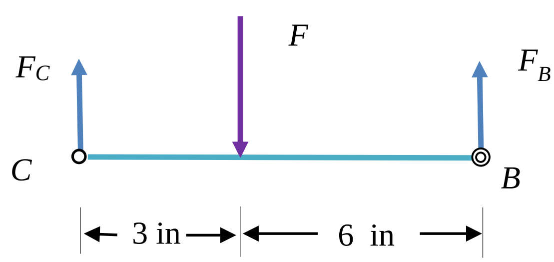
45. A force $F \sim N(1000, 100^2)$ lbf is applied to rod *BC* as shown in the figure. Rod *AB* has a round cross section and its yield strength is $S_y \sim N(5, 0.5^2)$ kpsi. If the maximum probability of failure is designed to be $p_f = 10^{-5}$, determine the minimum diameter of rod *AB* and select a preferred diameter. Note that *F* and S_y are independent.



Solution

Consider the free-body diagram of rod *BC*



According to the moment equilibrium of rod *BC* with respect to point *C*,

$$-F(3) + F_B(9) = 0$$

Solving for F_B yields

$$F_B = \frac{F}{3}$$

Thus the stress acting on the rod *AB* is

$$S = \frac{F_B}{A} = \frac{\frac{F}{3}}{\frac{\pi}{4}d^2} = \frac{4F}{3\pi d^2}$$

Thus the limit-state function is the actual stress subtracted from the yield strength. Failure occurs when $Y < 0$,

$$Y = g(\mathbf{X}) = S_y - S = S_y - \frac{4F}{3\pi d^2}$$

where $\mathbf{X}=(S_y, F)$.

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_\mathbf{X}) = \mu_{S_y} - \frac{4}{3\pi d^2}\mu_F = 5(10^3) - \frac{4}{3\pi d^2}(1000)$$

$$\sigma_Y = \sqrt{\left(\left.\frac{\partial g}{\partial S_y}\right|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_{S_y}\right)^2 + \left(\left.\frac{\partial g}{\partial F}\right|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_F\right)^2}$$

$$= \sqrt{\left(\sigma_{S_y}\right)^2 + \left(-\frac{4}{3\pi d^2}\sigma_F\right)^2}$$

$$= \sqrt{((0.5)(10^3))^2 + \left(-\frac{4}{3\pi d^2}(100)\right)^2}$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-\left(5(10^3) - \frac{4}{3\pi d^2}(1000)\right)}{\sqrt{((0.5)(10^3))^2 + \left(-\frac{4}{3\pi d^2}(100)\right)^2}}\right) = 10^{-5}$$

Thus

$$\frac{-\mu_Y}{\sigma_Y} = \frac{-\left(5(10^3) - \frac{4}{3\pi d^2}(1000)\right)}{\sqrt{((0.5)(10^3))^2 + \left(-\frac{4}{3\pi d^2}(100)\right)^2}} = \Phi^{-1}(10^{-5})$$

Solving for d yields

$$d = 0.404 \text{ in}$$

Thus $d = 0.5$ in can be used. **Ans.**