46. A cylindrical tube is subjected to an internal pressure $p \sim N(500, 50^2)$ psi. The tube has an inside diameter of $d_i = 10$ in and the allowable tangential stress is $S_a \sim N(12, 1^2)$ kpsi. If the maximum probability of failure is designed to be $p_f = 10^{-5}$, determine the minimum thickness of the tube using the theory of thin-walled vessels. Note that p and S_a are independent.

Solution

Applying the theory of thin-walled vessels, the maximum tangential stress is expressed as follows:

$$S_{max} = \frac{p(d_i + t)}{2t}$$

The limit-state function is the maximum tangential stress of the vessel subtracted from the allowable tangential stress. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = S_a - S_{max} = S_a - \frac{p(d_i + t)}{2t} = S_a - \frac{(10 + t)}{2t} p$$

where $\mathbf{X}$=(S_a, p).

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_{\mathbf{X}}) = \mu_{S_a} - \frac{(10 + t)}{2t}\mu_p = 12(10^3) - \frac{(10 + t)}{2t}(500)$$

$$\sigma_Y = \sqrt{\left(\left.\frac{\partial g}{\partial S_a}\right|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_{S_a}\right)^2 + \left(\left.\frac{\partial g}{\partial p}\right|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_p\right)^2} = \sqrt{\left(\sigma_{S_a}\right)^2 + \left(-\frac{(10 + t)}{2t}\sigma_p\right)^2}$$

$$= \sqrt{(1(10^3))^2 + \left(-\frac{(10 + t)}{2t}(50)\right)^2}$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-\left(12(10^3) - \frac{(10 + t)}{2t}(500)\right)}{\sqrt{(1(10^3))^2 + \left(-\frac{(10 + t)}{2t}(50)\right)^2}}\right) = 10^{-5}$$

Thus

$$\frac{-\mu_Y}{\sigma_Y} = \frac{-\left(12(10^3) - \frac{(10 + t)}{2t}(500)\right)}{\sqrt{(1(10^3))^2 + \left(-\frac{(10 + t)}{2t}(50)\right)^2}} = \Phi^{-1}(10^{-5})$$

Solving for t yields

$$t = 0.380 \text{ in}$$

Thus $t = 0.40$ in can be used. **Ans.**