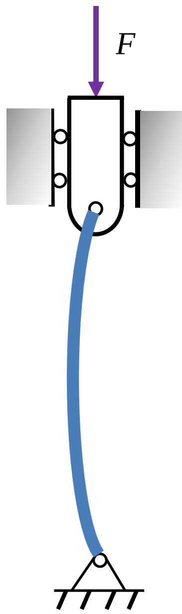
47. An axial force $F \sim N(8000,\ 400^2)$ lbf is applied to a round Euler column with a length of $l \sim N(2,\ 0.01^2)$ in. The ends of column are pined as shown in the figure. The modulus of elasticity is $E = 200$ kpsi. If the maximum probability of failure is designed to be $p_f = 10^{-5}$, determine the minimum diameter of the column using the First Order Second Moment Method. Note that F and l are independent.



Solution

According to the theory of Euler column, the critical load for unstable bending is

$$P_{cr} = \frac{C\pi^2 EI}{l^2} = \frac{C\pi^2 E}{l^2}\frac{\pi d^4}{64} = \frac{CE\pi^3 d^4}{64 l^2}$$

where $C = 1$, depending on the end conditions shown in the figure.

The limit-state function is the actual load of the column subtracted from the critical load. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = P_{cr} - F = \frac{CE\pi^3 d^4}{64 l^2} - F = \frac{200(10^3)\pi^3 d^4}{64} l^{-2} - F$$

where $\mathbf{X}$=(l, F).

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_\mathbf{X}) = \frac{200(10^3)\pi^3 d^4}{64}\mu_l^{-2} - \mu_F = \frac{200(10^3)\pi^3 d^4}{64} 2^{-2} - 8000$$

$$\sigma_Y = \sqrt{\left(\frac{\partial g}{\partial l}\bigg|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_l\right)^2 + \left(\frac{\partial g}{\partial F}\bigg|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_F\right)^2} = \sqrt{\left(\frac{200(10^3)\pi^3 d^4}{64}(-2)\mu_l^{-3}\sigma_l\right)^2 + (-\sigma_F)^2}$$

$$= \sqrt{\left(\frac{200(10^3)\pi^3 d^4}{64}(-2)2^{-3}(0.01)\right)^2 + (-400)^2}$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-\left(\frac{200(10^3)\pi^3 d^4}{64}2^{-2} - 8000\right)}{\sqrt{\left(\frac{200(10^3)\pi^3 d^4}{64}(-2)2^{-3}(0.01)\right)^2 + (-400)^2}}\right) = 10^{-5}$$

Thus

$$\frac{-\mu_Y}{\sigma_Y} = \frac{-\left(\frac{200(10^3)\pi^3 d^4}{64}2^{-2} - 8000\right)}{\sqrt{\left(\frac{200(10^3)\pi^3 d^4}{64}(-2)2^{-3}(0.01)\right)^2 + (-400)^2}} = \Phi^{-1}(10^{-5})$$

Solving for d yields

$$d = 0.797 \text{ in}$$

Thus $d = 0.80$ in can be used. **Ans.**