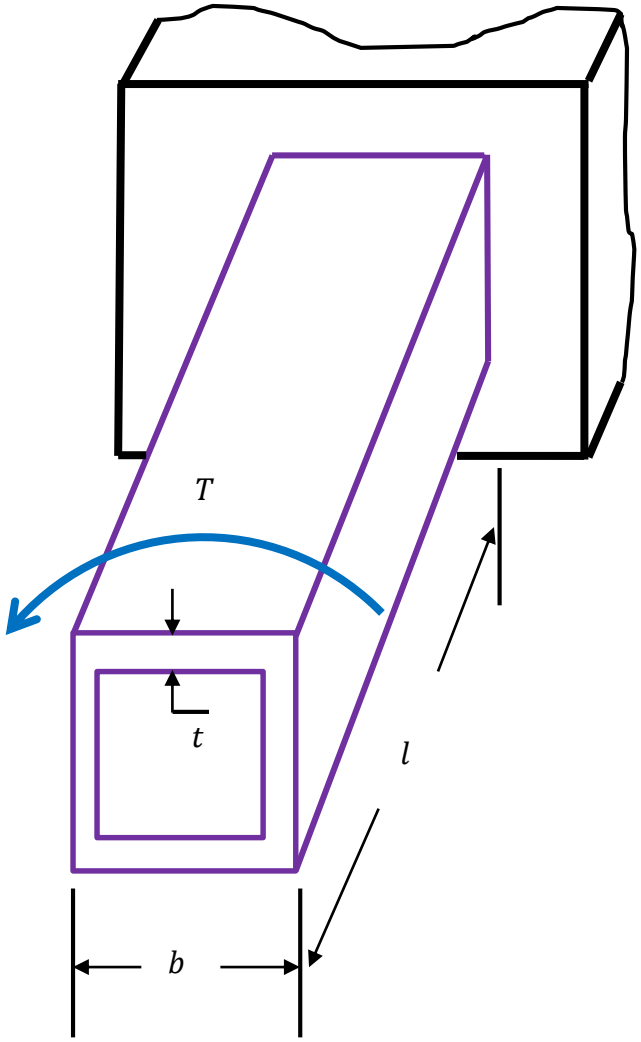
49. A steel thin-wall tube is subjected to a torque $T \sim N(10, 1^2)$ kN·m as shown in the figure. The tube is 400 mm long and has a square cross section with side length $b = 80$ mm. The shear modulus of the tube is $G = 80$ GPa. Assume that the allowable shear stress is $\tau_a \sim N(120, 12^2)$ MPa and the maximum probabilty of failure is $p_f = 10^{-5}$, estimate the minimum wall thickness of the tube using the First Order Second Moment Method. Note that T and τ_a are independent.



Solution

Based on the theory of closed thin-walled tube, the shear stress is

$$\tau = \frac{T}{2A_m t}$$

where A_m is the area enclosed by section median line

$$A_m = (b - t)(b - t) = (30(10^{-3}) - t)^2$$

The limit-state function is the actual shear stress subtracted from the allowable stress. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = \tau_a - \tau = \tau_a - \frac{T}{2A_m t} = \tau_a - \frac{1}{2(80(10^{-3}) - t)^2 t} T$$

where $\mathbf{X}=(\tau_a, T)$.

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_\mathbf{X}) = \mu_{\tau_a} - \frac{1}{2(80(10^{-3}) - t)^2 t}\mu_T = 120(10^6) - \frac{1}{2(80(10^{-3}) - t)^2 t}(10)(10^3)$$

$$\sigma_Y = \sqrt{\left(\frac{\partial g}{\partial \tau_a}\Big|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_{\tau_a}\right)^2 + \left(\frac{\partial g}{\partial T}\Big|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_T\right)^2}$$

$$= \sqrt{(\sigma_{\tau_a})^2 + \left(-\frac{1}{2(80(10^{-3}) - t)^2 t}\sigma_T\right)^2}$$

$$= \sqrt{\left(12(10^6)\right)^2 + \left(-\frac{1}{2(80(10^{-3}) - t)^2 t}(1)(10^3)\right)^2}$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-\left(120(10^6) - \frac{1}{2(80(10^{-3}) - t)^2 t}(10)(10^3)\right)}{\sqrt{\left(12(10^6)\right)^2 + \left(-\frac{1}{2(80(10^{-3}) - t)^2 t}(1)(10^3)\right)^2}}\right) = 10^{-5}$$

Thus

$$\frac{-\mu_Y}{\sigma_Y} = \frac{-\left(120(10^6) - \frac{1}{2(80(10^{-3}) - t)^2 t}(10)(10^3)\right)}{\sqrt{\left(12(10^6)\right)^2 + \left(-\frac{1}{2(80(10^{-3}) - t)^2 t}(1)(10^3)\right)^2}} = \Phi^{-1}(10^{-5})$$

Solving for t yields

$$t = 26.7 \text{ mm}$$

Thus $t = 28$ mm can be used. **Ans.**