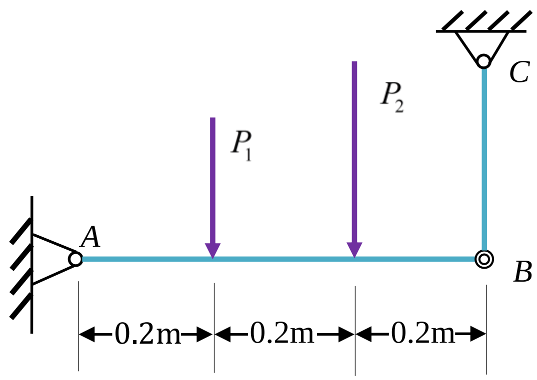
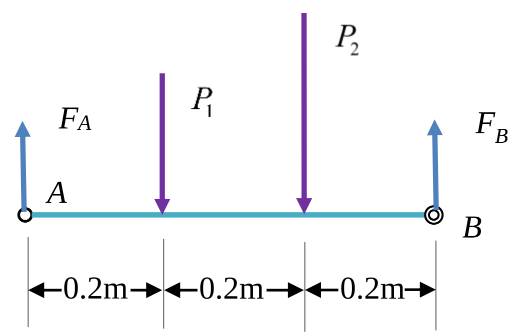
5. If $P_1 \sim N(10,1^2)\,\text{kN}$ and $P_2 \sim N(15,2^2)\,\text{kN}$, what is the distribution of the stress acting on rod *BC*? The diameter of rod *BC* is 0.05 m. Assume P_1 and P_2 are independent.



Solution

Consider the force equilibrium of rod *AB* as shown in the figure below



According to the moment equilibrium of rod *AB* with respect to point *A*,

$$M_A + M_1 + M_2 + M_B = 0 \tag{1}$$

where M_A is the moment resulted from F_A, M_1 is the moment resulted from F_1, M_2 is the moment resulted from F_2, and M_B is the moment resulted from F_B.

Then

$$0 + P_1(0.2) + P_2(0.4) - F_B(0.6) = 0 \tag{2}$$

Solving for F_B yields

$$F_B = \frac{1}{3}P_1 + \frac{2}{3}P_2 \tag{3}$$

Thus the stress acting on rod *BC* is

$$S = \frac{F_B}{A} = \frac{F_B}{\frac{\pi}{4}D^2} = \frac{4F_B}{\pi D^2} \tag{4}$$

where *A* is the cross-sectional area of rod *BC*, and *D* is the diameter of rod *BC*.

Combining Eqs. (3) and (4) yields

$$S = \frac{4}{3\pi D^2}P_1 + \frac{8}{3\pi D^2}P_2 \tag{5}$$

Because P_1 and P_2 are independently and normally distributed, their linear sum, S, is also normally distributed. The mean and standard deviation of S are given by

$$\mu_S = \frac{4}{3\pi D^2}\mu_{P_1} + \frac{8}{3\pi D^2}\mu_{P_2} = \frac{4}{3\pi(0.05)^2}(10000) + \frac{8}{3\pi(0.05)^2}(15000) = 6.79 \text{ MPa}$$

$$\sigma_S = \sqrt{\left(\frac{4}{3\pi D^2}\sigma_{P_1}\right)^2 + \left(\frac{8}{3\pi D^2}\sigma_{P_2}\right)^2} = \sqrt{\left(\frac{4}{3\pi(0.05)^2}(1000)\right)^2 + \left(\frac{8}{3\pi(0.05)^2}(2000)\right)^2} = 0.70 \text{ MPa}$$

So the distribution of the stress acting on rod *BC* is $S \sim N(6.79, 0.70^2)$ MPa **Ans.**