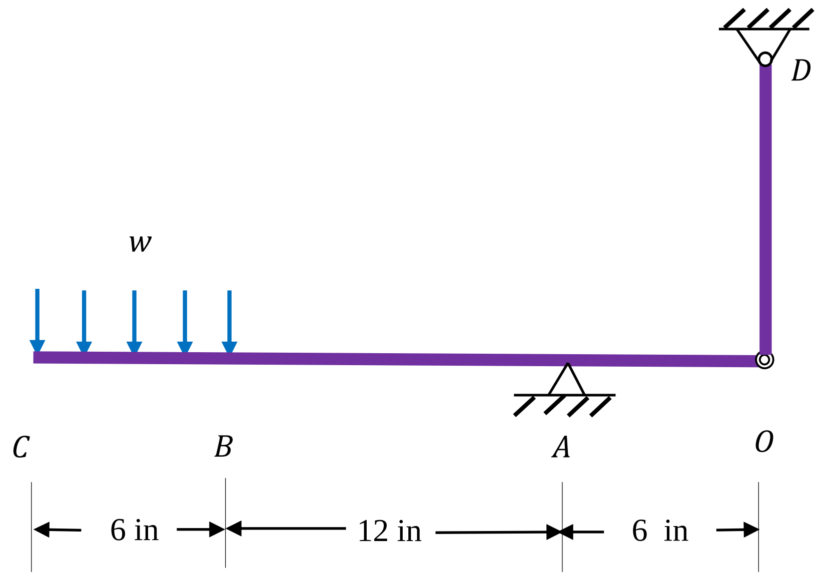
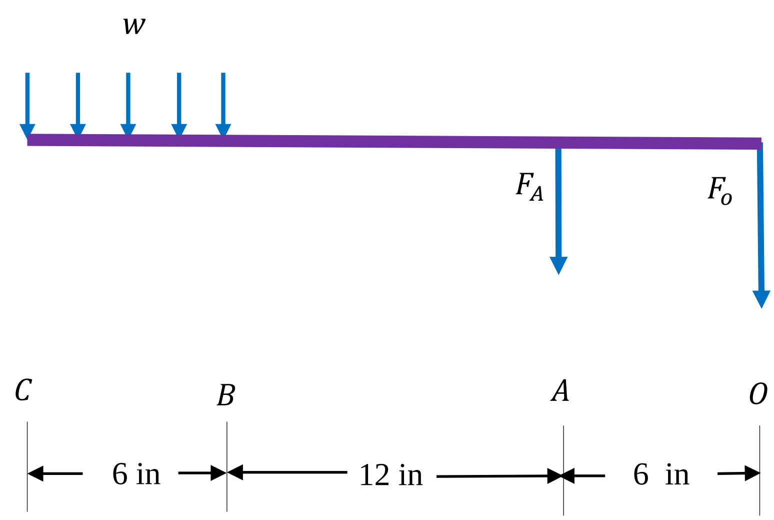
50. A rod *OC* is subjected to a uniform load of $w \sim N(100,\ 10^2)$ lbf/in as shown in the figure. The yield strength of rod *OD* is $S_y \sim N(20, 2^2)$ kips. If the maximum probability of failure is designed to be $p_f = 10^{-5}$, determine the minimum diameter of rod *OD* using the First Order Second Moment Method and then select a preferred one. Note that w and S_y are independent.



Solution

Consider the free body diagram of rod *OC* shown in the figure.



Based on the moment equilibrium of rod *OC* with respect to point *A*,

$$F_o l_{OA} - w l_{BC}(l_{AB} + \frac{l_{BC}}{2}) = 0$$

Solving for F_0 yields

$$F_o = \frac{w l_{BC}(2l_{AB} + l_{BC})}{2l_{OA}}$$

Thus the compressed stress acting on rod *OD* is given by

$$S = \frac{F_o}{A} = \frac{\frac{w l_{BC}(2l_{AB} + l_{BC})}{2l_{OA}}}{\frac{\pi}{4}d^2} = \frac{2l_{BC}(2l_{AB} + l_{BC})}{\pi l_{OA} d^2} w$$

The limit-state function is the actual compressed stress subtracted from the yield strength. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = S_y - S = S_y - \frac{2l_{BC}(2l_{AB} + l_{BC})}{\pi l_{OA} d^2} w = S_y - \frac{2(6)(2(12) + 6)}{\pi(6)d^2} w$$

$$= S_y - \frac{60}{\pi d^2} w$$

where $\mathbf{X}$=(S_y, w).

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_\mathbf{X}) = \mu_{S_y} - \frac{60}{\pi d^2}\mu_w = 20(10^3) - \frac{60}{\pi d^2}(100)$$

$$\sigma_Y = \sqrt{\left(\frac{\partial g}{\partial S_y}\bigg|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_{S_y}\right)^2 + \left(\frac{\partial g}{\partial w}\bigg|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_w\right)^2}$$

$$= \sqrt{\left(\sigma_{S_y}\right)^2 + \left(-\frac{60}{\pi d^2}\sigma_w\right)^2}$$

$$= \sqrt{\left(2(10^3)\right)^2 + \left(-\frac{60}{\pi d^2}(10)\right)^2}$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-\left(20(10^3) - \frac{60}{\pi d^2}(100)\right)}{\sqrt{\left(2(10^3)\right)^2 + \left(-\frac{60}{\pi d^2}(10)\right)^2}}\right) = 10^{-5}$$

Thus

$$\frac{-\mu_Y}{\sigma_Y} = \frac{-\left(20(10^3) - \frac{60}{\pi d^2}(100)\right)}{\sqrt{\left(2(10^3)\right)^2 + \left(-\frac{60}{\pi d^2}(10)\right)^2}} = \Phi^{-1}(10^{-5})$$

Solving for d yields

$$d = 0.43 \text{ in}$$

Thus $d = 0.5$ in can be used. **Ans.**