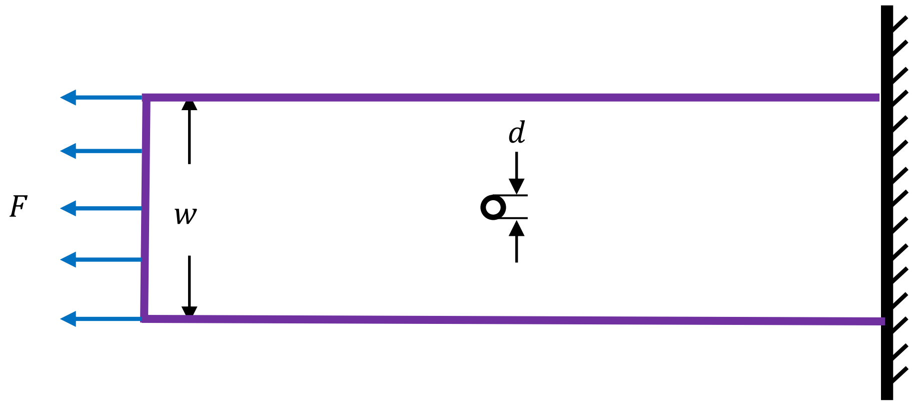
51. A brittle thin plate is subjected to a random force $F \sim N(20,\ 2^2)$ kN. The plate has a width of $w = 100$ mm and a hole in the center with a diameter of $d = 10$ mm. The yield strength of the plate is $S_y \sim N(500, 5^2)$ MPa. Assume the stress concentration factor is $K_t = 2.7$ and the maximum probability of failure is designed to be $p_f = 10^{-5}$, estimate the minimum thickness of the plate using the First Order Second Moment Method.



Solution

The nominal stress is

$$S_n = \frac{F}{A} = \frac{F}{(w-d)t}$$

With the effect of stress concentration, the maximum stress is given by

$$S_{\max} = K_t S_n = \frac{K_t}{(w-d)t} F$$

The limit-state function is the maximum stress subtracted from the yield strength. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = S_y - S_{\max} = S_y - \frac{K_t}{(w-d)t} F = S_y - \frac{2.7}{(100(10^{-3}) - 10(10^{-3}))t} F$$

where **X**=(S_y, F).

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_\mathbf{X}) = \mu_{S_y} - \frac{2.7}{\left(100(10^{-3}) - 10(10^{-3})\right)t} \mu_F$$

$$= 500(10^6) - \frac{2.7}{(100(10^{-3}) - 10(10^{-3}))t} (20)(10^3)$$

$$\sigma_Y = \sqrt{\left(\left.\frac{\partial g}{\partial S_y}\right|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_{S_y}\right)^2 + \left(\left.\frac{\partial g}{\partial F}\right|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_F\right)^2}$$

$$= \sqrt{\left(\sigma_{S_y}\right)^2 + \left(-\frac{2.7}{\left(100(10^{-3}) - 10(10^{-3})\right)t}\sigma_F\right)^2}$$

$$= \sqrt{(5(10^6))^2 + \left(\frac{2.7}{\left(100(10^{-3}) - 10(10^{-3})\right)t}(2)(10^3)\right)^2}$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-\left(500(10^6) - \frac{2.7}{(100(10^{-3}) - 10(10^{-3}))t}(20)(10^3)\right)}{\sqrt{(5(10^6))^2 + \left(\frac{2.7}{\left(100(10^{-3}) - 10(10^{-3})\right)t}(2)(10^3)\right)^2}}\right) = 10^{-5}$$

Thus

$$\frac{-\mu_Y}{\sigma_Y} = \frac{-\left(500(10^6) - \frac{2.7}{(100(10^{-3}) - 10(10^{-3}))t}(20)(10^3)\right)}{\sqrt{(5(10^6))^2 + \left(\frac{2.7}{\left(100(10^{-3}) - 10(10^{-3})\right)t}(2)(10^3)\right)^2}} = \Phi^{-1}(10^{-5})$$

Solving for t yields

$$t = 1.72 \text{ mm}$$

Thus $t = 1.8$ mm can be used. **Ans.**