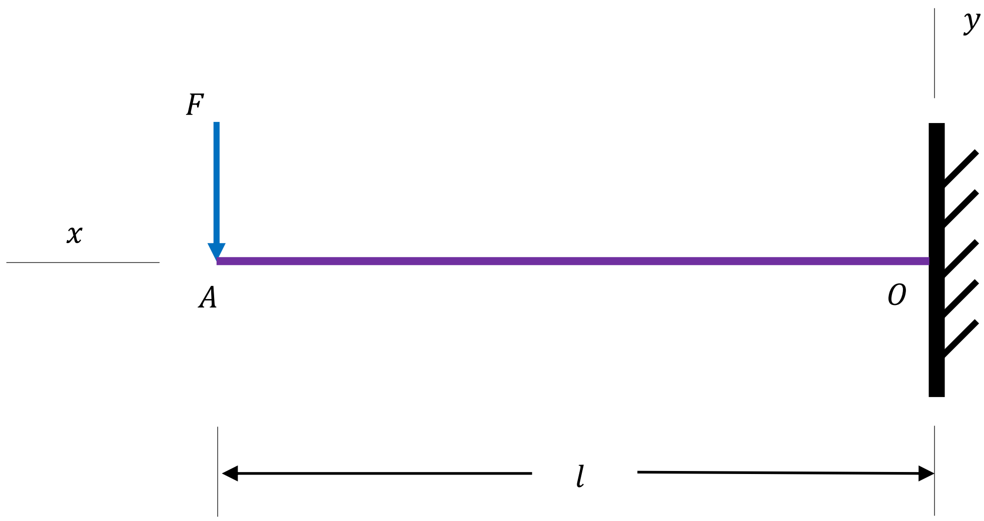
52. A force $F \sim N(3000,\ 300^2)$ N is applied to a cantilevered tube as shown in the figure. The tube has an outside diameter of $d_O = 50$ mm and inside diameter of d_i. The lenghth of the tube is $l \sim N(1300, 0.1^2)$ mm and the modulus of elasticity is $E = 80$ GPa. Assume that the allowable transverse deflection is $y_a = 130$ mm and the maximum probability of failure is designed to be $p_f = 10^{-5}$, estimate the inside diameter of the tube. Note that the maximum deflection is $y_{\max} = \frac{Fl^3}{3EI}$



Solution

The maximum deflection happens at O and is given by

$$y_{\max} = \frac{Fl^3}{3EI} = \frac{Fl^3}{3E\frac{\pi}{64}(d_o^4 - d_i^4)} = \frac{64Fl^3}{3\pi E(d_o^4 - d_i^4)} = \frac{64}{3\pi(80)(10^9)\left(\left(50(10^{-3})\right)^4 - d_i^4\right)}Fl^3$$

The limit-state function is the maximum deflection subtracted from allowable transverse deflection. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = y_a - y_{\max} = y_a - \frac{64Pl^3}{3\pi E(d_o^4 - d_i^4)} = 130(10^{-3}) - \frac{64}{3\pi(80)(10^9)\left(\left(50(10^{-3})\right)^4 - d_i^4\right)}Fl^3$$

where $\mathbf{X}=(F, l)$.

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_\mathbf{X}) = 130(10^{-3}) - \frac{64}{3\pi(80)(10^9)\left(\left(50(10^{-3})\right)^4 - d_i^4\right)}\mu_F\mu_l^3$$

$$= 130(10^{-3}) - \frac{64}{3\pi(80)(10^9)\left(\left(50(10^{-3})\right)^4 - d_i^4\right)} 3000\left(1300(10^{-3})\right)^3$$

$$\sigma_Y = \sqrt{\left(\left.\frac{\partial g}{\partial F}\right|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_P\right)^2 + \left(\left.\frac{\partial g}{\partial l}\right|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_l\right)^2}$$

$$= \sqrt{\begin{array}{c}\left(-\frac{64}{3\pi(80)(10^9)\left(\left(50(10^{-3})\right)^4 - d_i^4\right)} \mu_l^3 \sigma_F\right)^2 \\ + \left(-\frac{64}{3\pi(80)(10^9)\left(\left(50(10^{-3})\right)^4 - d_i^4\right)} (3)(\mu_F \mu_l^2) \sigma_l\right)^2\end{array}}$$

$$= \sqrt{\begin{array}{c}\left(-\frac{64}{3\pi(80)(10^9)\left(\left(50(10^{-3})\right)^4 - d_i^4\right)} \left(1300(10^{-3})\right)^3 (300)\right)^2 \\ + \left(-\frac{64}{3\pi(80)(10^9)\left(\left(50(10^{-3})\right)^4 - d_i^4\right)} (3)\left(3000\left(1300(10^{-3})\right)^2\right)(0.1(10^{-3}))\right)^2\end{array}}$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = 10^{-5}$$

Thus

$$\frac{-\mu_Y}{\sigma_Y} = -\frac{130(10^{-3}) - \frac{64}{3\pi(80)(10^9)\left(\left(50(10^{-3})\right)^4 - d_i^4\right)} 3000\left(1300(10^{-3})\right)^3}{\sqrt{\begin{array}{c}\left(-\frac{64}{3\pi(80)(10^9)\left(\left(50(10^{-3})\right)^4 - d_i^4\right)} \left(1300(10^{-3})\right)^3 (300)\right)^2 \\ + \left(-\frac{64}{3\pi(80)(10^9)\left(\left(50(10^{-3})\right)^4 - d_i^4\right)} (3)\left(3000\left(1300(10^{-3})\right)^2\right)(0.1(10^{-3}))\right)^2\end{array}}}$$

$$= \Phi^{-1}(10^{-5})$$

Solving for d_i yields

$$d_i = 18.3 \text{ mm}$$

Thus $d_i = 18$ mm can be used. **Ans.**