53. A tension rod has a length of $l \sim N(2,\ 0.002^2)$ m and a yield strength of $S_y \sim N(60,\ 6^2)$ MPa. The modulus of elasticity is $E = 80$ GPa. If the maximum probability of failure is designed to be $p_f = 10^{-5}$, determine the allowable axial elongation using the First Order Second Moment Method.

Solution

Based on Hooke's law, the tensile stress is given by

$$S = E\epsilon = E\frac{\delta_a}{l}$$

where ϵ is the stain.

The limit-state function is the actual tensile stress subtracted from the yield strength. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = S_y - S = S_y - \frac{E}{l}\delta_a = S_y - \frac{E}{l}\delta_a$$

where $\mathbf{X}$=(S_y, l).

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_\mathbf{X}) = \mu_{S_y} - \frac{E}{\mu_l}\delta_a = 60(10^6) - \frac{80(10^9)}{2}\delta_a$$

$$\sigma_Y = \sqrt{\left(\frac{\partial g}{\partial S_y}\bigg|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_{S_y}\right)^2 + \left(\frac{\partial g}{\partial l}\bigg|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_l\right)^2}$$

$$= \sqrt{\left(\sigma_{S_y}\right)^2 + \left(\frac{E}{\mu_l^2}\delta_a\sigma_l\right)^2}$$

$$= \sqrt{(6(10^6))^2 + \left(\frac{80(10^9)}{2^2}\delta_a(0.002)\right)^2}$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-\left(60(10^6) - \frac{80(10^9)}{2}\delta_a\right)}{\sqrt{(6(10^6))^2 + \left(\frac{80(10^9)}{2^2}\delta_a(0.002)\right)^2}}\right) = 10^{-5}$$

Thus

$$\frac{-\mu_Y}{\sigma_Y} = \frac{-\left(60(10^6) - \frac{80(10^9)}{2}\delta_a\right)}{\sqrt{(6(10^6))^2 + \left(\frac{80(10^9)}{2^2}\delta_a(0.002)\right)^2}} = \Phi^{-1}(10^{-5})$$

Solving for δ_a yields

$$\delta_a = 0.86\text{ mm}$$

Ans.