54. A tensile force $F \sim N(18, 1^2)$ kN is applied to a rod. The rod has a diameter of $d \sim N(10, 0.1^2)$ mm and a length of $l \sim N(50, 0.1^2)$ mm. The modulus of elasticity is $E = 80$ GPa and the poission ratio is $\nu = 0.29$. If the allowable change in rod diameter is $\Delta d_a = 0.01$ mm, estimate the probability of failure using the First Order Second Moment Method.

Solution

The tensile stress is

$$S = \frac{F}{A} = \frac{F}{\frac{\pi d^2}{4}} = \frac{4F}{\pi d^2}$$

And the corresponding axial strain is given by

$$\epsilon_a = \frac{S}{E} = \frac{4F}{\pi d^2 E}$$

Then the lateral strain is

$$\epsilon_l = \nu \epsilon_a = \frac{4F\nu}{\pi d^2 E}$$

Thus the change in rod diameter is

$$\Delta d = \epsilon_l d = \frac{4\nu}{\pi E} \frac{F}{d}$$

The limit-state function is the actual change in rod diameter subtracted from the allowable one. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = \Delta d_a - \Delta d = \Delta d_a - \frac{4\nu}{\pi E} \frac{F}{d} = 0.01(10^{-3}) - 4.62(10^{-12}) \frac{F}{d}$$

where $\mathbf{X} = (F, d)$.

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_{\mathbf{X}}) = \Delta d_a - 4.62(10^{-12}) \frac{\mu_F}{\mu_d} = 0.01(10^{-3}) - 4.62(10^{-12}) \frac{18(10^3)}{10(10^{-3})} = 1.69(10^{-6}) \text{ mm}$$

$$\sigma_Y = \sqrt{\left(\frac{\partial g}{\partial F}\bigg|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_P \right)^2 + \left(\frac{\partial g}{\partial d}\bigg|_{\boldsymbol{\mu}_{\mathbf{X}}} \sigma_d \right)^2}$$

$$= \sqrt{\left(-4.62(10^{-12}) \frac{1}{\mu_d} \sigma_F \right)^2 + \left(4.62(10^{-12}) \frac{\mu_F}{\mu_d^2} \sigma_d \right)^2}$$

$$= \sqrt{\left(-4.62(10^{-12})\frac{1}{10(10^{-3})}1(10^{3})\right)^2 + \left(4.62(10^{-12})\frac{18(10^{3})}{\left(10(10^{-3})\right)^2}(0.1)(10^{-3})\right)^2}$$

$$= 4.69(10^{-7})\ \text{mm}$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-1.69(10^{-6})}{4.69(10^{-7})}\right) = 1.54(10^{-4})$$ **Ans.**