55. A grinding wheel has a weight of $m \sim N(1,\ 0.1^2)$ kg, a diameter of $d_o = 250$ mm, and a bore with a diameter of $d_i = 20$ mm. The thickness of the wheel is $t = 5$ mm, and the Poisson's ration is $\nu = 0.20$. If the allowable tangential stress is $S_a \sim N(3,\ 0.3^2)$ MPa and the maximum probability of failure is designed to be $p_f = 10^{-5}$, determine the maximum working speed of the grinding wheel using the theory of rotating rings.

Solution

The outside radius and inside radius of the wheel are

$$r_o = \frac{d_o}{2} = \frac{250}{2} = 125\ \text{ mm}$$

$$r_i = \frac{d_i}{2} = \frac{20}{2} = 10\ \text{ mm}$$

Thus the mass density of the wheel is

$$\rho = \frac{m}{V} = \frac{m}{\frac{\pi}{4}(d_o^2 - d_i^2)t} = 4100.61m$$

And the angular velocity of the wheel is

$$\omega = \frac{2\pi n}{60} = 0.105n$$

According to the theory of rotating rings, the maximum tangential stress is

$$\begin{aligned} S_{max} &= \rho\omega^2\left(\frac{3+\nu}{8}\right)\left(r_i^2 + r_o^2 + \frac{r_i^2 r_o^2}{r_i^2} - \frac{1+3\nu}{3+\nu}r_i^2\right) \\ &= 4100.61m(0.105n)^2\left(\frac{3+0.20}{8}\right)\left(0.01^2 + 0.125^2 + 0.125^2 - \frac{1+3(0.20)}{3+0.20}(0.01^2)\right) \\ &= 0.566mn^2 \end{aligned}$$

The limit-state function is the maximum tangential stress of the wheel subtracted from the allowable tangential stress. Failure occurs when $Y < 0$.

$$Y = g(\mathbf{X}) = S_a - S_{max} = S_a - 0.566mn^2$$

where $\mathbf{X}$=(S_a, m).

Using FOSM, we have

$$\mu_Y = g(\boldsymbol{\mu}_\mathbf{X}) = \mu_{S_a} - 0.566\mu_m n^2$$

$$\sigma_Y = \sqrt{\left(\frac{\partial g}{\partial S_a}\bigg|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_{S_a}\right)^2 + \left(\frac{\partial g}{\partial m}\bigg|_{\boldsymbol{\mu}_\mathbf{X}} \sigma_m\right)^2} = \sqrt{\left(\sigma_{S_a}\right)^2 + (-0.566 n^2 \sigma_m)^2}$$

The probability of failure is then given by

$$p_f = \Phi\left(\frac{-\mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{-\left(\mu_{S_a} - 0.566\mu_m n^2\right)}{\sqrt{\left(\sigma_{S_a}\right)^2 + (-0.566 n^2 \sigma_m)^2}}\right) = 10^{-5}$$

Thus

$$\Phi^{-1}(10^{-5}) = \frac{-\left(\mu_{S_a} - 0.566\mu_m n^2\right)}{\sqrt{\left(\sigma_{S_a}\right)^2 + (-0.566 n^2 \sigma_m)^2}}$$

$$= \frac{-\left(120(10^6) - \dfrac{152.8(1)(10^3)}{\pi\left(80(10^{-3})\right)^3 n}\right)}{\sqrt{\left(12(10^6)\right)^2 + \left(\dfrac{152.8(1)(10^3)}{\pi\left(80(10^{-3})\right)^4 n}(-3)\right)^2 \left((0.8)(10^{-3})\right)^2}}$$

Solving for n yields

$$n = 5023 \text{ rpm}$$

Ans.